

1 **A Comparison of Mixture Fraction Approaches for Coal Combustion Simulations**

2 Andrew P. Richards, David O. Lignell, and Thomas H. Fletcher*

3 Chemical Engineering Department, Brigham Young University

4 Provo, Utah, USA 84602

5
6 Abstract

7 Simulations of industrial-scale coal combustion systems often use simplifying
8 assumptions of gas-phase chemical reactions due to computational considerations and
9 complications of turbulence-chemistry interactions. One common method is to use a mixture
10 fraction approach combined with equilibrium chemistry, assuming that reactions are mixing-
11 limited. Other assumptions may be that the gas from the coal may be assumed to have the same
12 composition as the coal, or a simple compound like benzene, or that the char is assumed to have
13 the chemical composition and heating value of graphite. This paper shows that these simplifying
14 assumptions lead to inaccuracies in equilibrium temperature and gas-phase mole fractions, with
15 the peak CO and H₂O mole fractions varying by up to 20 mol% and the peak equilibrium
16 temperature varying by 200 K, depending on the initial fuel chosen. In addition, this paper
17 explores the variability, accuracy, and utility of one-, two-, and three-fuel mixture fraction
18 approaches to treat char, tar, and light gases.

19 Keywords:

20 Coal; Pyrolysis; Multiple Mixture Fractions; Equilibrium Modeling; Cantera; NASA-CEA

21 *Corresponding Author

24 1. Introduction

25 Large-scale simulations of coal combustion systems have become very important both to
26 investigate new technologies and to increase understanding of coal combustion phenomena.
27 Flexible and accurate submodels are necessary for the overall accuracy and utility of the
28 simulation. Gas-phase reactions are an integral part of coal combustion simulations, and
29 influence the bulk gas composition, local gas temperature, and can have implications on fluid
30 flow, heat transfer, and mass transfer [1]. Simplifying assumptions are often used to decrease
31 computational complexity, especially if computational resources are limited. Advances in
32 computer technology allow for larger and more complex problems to be simulated, and larger
33 and more complex submodels are becoming increasingly used to decrease overall error and
34 uncertainty in these large-scale simulations [2, 3].

35 Coal is very complex, and the structure and composition change even in coals from the
36 same seam [4]. Because of this complexity, many of the simplifying assumptions in gas-phase
37 equilibrium of coal systems have dealt with simplifying coal-based fuel compositions,
38 particularly the compositions of pyrolysis products (char, tar, and light gases). Frequently coal
39 char is modeled as pure carbon with the thermochemical properties of either graphite [5-12] or of
40 the parent coal [13-16]. This is perhaps useful in heterogeneous char reactions since oxidation
41 and gasification reactions primarily target the carbon in the char. However, this approach falls
42 short in calculating other combustion products (i.e., H₂O and other hydrogen-based products),
43 since the char retains some hydrogen and other atoms such as oxygen, nitrogen, and sulfur [17,
44 18].

45 Other simulations have simplified the compositions and energies of the volatile species,
46 often using one or more of several simple hydrocarbons in the place of the more complex tars

47 and other volatile gases. Some simulations use methane in the place of the total volatiles [7, 12,
48 14, 19-29] or other simple C₂ hydrocarbons [30]. Other approaches use a mix of common
49 pyrolysis gases (CH₄, C₆H₆, CO, CO₂, H₂, N₂, H₂S, etc.) in variable quantities to ensure
50 elemental balance between the original coal and gaseous species [20, 23, 31, 32]. Some even use
51 constant generic hydrocarbon estimates (C_xH_yO_z) to better approximate volatile species [10, 13,
52 15, 24, 29, 33-35], often approximated using the composition of the original coal. While these
53 simplified approaches may be appropriate for smaller-scale or simplified calculations, they may
54 be inadequate for accurate large-scale simulations, especially since inaccuracies in CO₂ and H₂O
55 gas compositions greatly affect other parts of the coal combustion system like radiative heat
56 transfer [36]. Volatile gases released by coal pyrolysis reactions contain many of these simple
57 hydrocarbons, but the total volatiles tend to be much more complex, particularly the tar species
58 [18, 37]. This paper details attempts to quantify uncertainty and error in using these simplified
59 approaches by comparing several of the simple coal surrogate gases to measured coal-based
60 fuels.

61 Coal products change composition depending on a variety of conditions, including local
62 gas temperature, particle residence times, and heating rate [17, 38, 39]. Compositions and other
63 properties also change as a particle moves through the hot gas environment [40]. To help with
64 mixing and variable fuel compositions, large-scale simulations frequently use one of several
65 different modeling approaches, including flamelet and progress variables [11, 41-44], mixture
66 fractions [3, 45-50], full equilibrium [51], approximations to equilibrium such as the water-gas
67 shift reaction [52], detailed kinetic mechanisms of simpler hydrocarbon fuels such as GRI3.0
68 [21, 22, 41, 43, 53, 54], or even a combination of several different approaches [11, 27, 28, 44,
69 46].

70 This paper compares three different levels of mixture fraction analyses, including one,
71 two, and three fuel mixture fractions for coal combustion scenarios. The one-mixture fraction
72 approach assumes all mass originating in the coal has the same elemental composition and
73 heating value as the parent coal and is the most common mixture fraction approach (e.g., [1, 3,
74 51, 55]). The two-mixture fraction approach treats volatiles and char compositions and heating
75 values separately, and was explored by Flores and Fletcher [50] in simulations of three
76 laboratory-scale coal combustors. In that study, the one- and two-mixture fraction approaches
77 showed differences in calculated temperatures and species concentrations near the burner,
78 especially with NO predictions. The three-mixture fraction approach treats light gas, tar, and char
79 separately. These three approaches are compared in this paper using equilibrium calculations,
80 rather than performing simulations of pilot- or industrial-scale furnaces. This simplified approach
81 allows focus on the chemical state spaces, which can be widely explored, and which would be
82 encompassed in specific furnace simulations.

83

84 1.1 Mixture Fractions

85 This section introduces the mixture fraction methods used in this analysis, beginning with
86 a brief discussion on mixture fractions in general. This discussion includes some of the
87 differences in applying mixture fractions to coal-based fuels as opposed to other simple
88 hydrocarbon fuels. Next, each level of mixture fraction comparison is detailed, starting with the
89 one-mixture fraction comparison, followed by the two-mixture fraction comparison. Finally, the
90 three-mixture fraction comparison is discussed.

91 At its most simple definition, a mixture fraction analysis involves defining a reacting
92 system (e.g., combustion of a hydrocarbon fuel with air) into two types of streams—one that

93 includes all fuels and another that includes the oxidizing gases. In most gas-phase combustion
94 reactions of simple hydrocarbon fuels, mixture fractions can be considered “conserved scalars,”
95 which means that the mixture fraction does not change with reacting conditions. Conserved
96 scalars simplify reacting flow calculations and can be anything from elemental mass fractions to
97 enthalpies of the different reacting streams, as long as it is a stream property that remains the
98 same throughout the reacting system [56]. This approach is a simple way to track material when
99 it mixes and reacts to equilibrium.

100 Mixture fractions in coal-based systems are not usually considered conserved scalars
101 because the mixture fractions typically include a separate source term to describe the addition of
102 material into the gas phase from a solid coal particle [1, 50]. This source term is necessary
103 because coal combustion is not a homogeneous reaction that occurs only in the gas phase, but
104 instead involves a complex reaction between a solid coal particle and other background gases
105 [4]. While using a source term is helpful in describing the physical processes of a coal
106 combustion reaction, it can complicate transport and reaction equations, making a mixture
107 fraction description of coal combustion difficult to incorporate into large-scale simulations.

108 The mixture fraction comparisons used in this paper are slightly different from those used
109 traditionally in simple hydrocarbon reactions and from previous work on coal-gas mixture
110 fractions. For this reason, a detailed derivation of the mixture fraction and other necessary
111 equations is detailed here. While there are multiple ways of defining a mixture fraction,
112 including the widely used Bilger’s mixture fraction [57-59], this analysis explores two main
113 types of mixture fractions. The first is a more traditional form of mixture fraction (named here as
114 a “component” or “fuel” mixture fraction and labeled as f_i to avoid confusion with other mixture
115 fractions) which is the mass fraction of material originating in each stream. This component

116 mixture fraction, f_i , is calculated by dividing the mass of one fuel stream by the total mass of the
117 mixture [56], shown in general form in Equation 1:

118

$$119 \quad f_i = \frac{M_i}{M_{ox} + \sum_{i=1}^N M_i} \quad (1)$$

120

121 where f_i is the component mixture fraction of component i (coal, char, tar, oxidizer, etc.), M_i is
122 the total mass of component i , M_{ox} is the total mass of the oxidizer component (e.g., air), and N
123 indicates the total number of fuel components. The sum of all component mixture fractions adds
124 to one, and the total number of fuel mixture fractions to completely describe a reacting system is
125 $N - 1$.

126 The component mixture fraction is commonly used to describe a system of one fuel
127 stream and one oxidizer stream—otherwise known as a one-mixture fraction system. While one-
128 mixture fraction systems are common with many types of gas-phase combustion applications,
129 this type of component mixture fraction can be extended to two, three, or even more [60].

130 The second type of mixture fraction is called the “elemental” mixture fraction and labeled
131 as Z_j to distinguish from the component mixture fraction. This type of mixture fraction describes
132 the total mass fraction of each element in the reacting system [59, 61]. The elemental mixture
133 fraction is shown in Equation 2:

134

$$135 \quad Z_j = W_j \sum_{i=1}^N \frac{a_{ij} Y_i}{W_i} \quad (2)$$

136

137 where Z_j is the elemental mixture fraction of element j (i.e. carbon, hydrogen, etc.), W_j is the
138 molecular weight of element j , W_i is the molecular weight of component i (including oxidizer

139 and all fuels), Y_i is the mass fraction of component i (the component mixture fraction), and a_{ij} is
140 the number of atoms of element j in component i (for example, a sample fuel of benzene, C_6H_6
141 would have an a_{ij} of 6 for both carbon and hydrogen). In most simple hydrocarbon combustion
142 systems, the total number of primary elements is four (CHON), however, in coal combustion
143 applications, sulfur is present, leading to five total elemental mixture fractions to describe the
144 primary organic elements (CHONS) present in the fuel and oxidizer streams. Like the component
145 mixture fraction, the elemental mixture fractions sum to one.

146 Another useful parameter in combustion modeling is the equivalence ratio [56] (shown in
147 Equation 3), which relates the fuel-to-oxidizer ratio of a mixture to the fuel's stoichiometric fuel-
148 to-oxidizer ratio.

149

$$150 \quad \phi_i = \frac{\frac{n_i}{n_{ox}}}{\left(\frac{n_i}{n_{ox}}\right)_{stoich}} \quad (3)$$

151

152 where ϕ_i is the equivalence ratio of fuel i , n_i is the number of moles of fuel i , n_{ox} is the moles
153 of oxidizer (i.e., air), and the subscript *stoich* is the oxidizer-to-fuel ratio at stoichiometric
154 proportions. The equivalence ratio or component mixture fraction can be used to determine the
155 total moles of oxidizer in the system if the amount of fuel is fixed to a constant value.

156

157 1.1.1 One-Mixture Fraction Comparison

158 A one-mixture fraction method divides a combustion mixture into two streams—one fuel
159 stream and one oxidizer stream. This is a common method used in simple hydrocarbon
160 combustion applications and has also been applied to coal-gas mixtures [1]. The one-mixture

161 fraction comparison is the simplest of the three detailed here. In coal systems, one mixture
 162 fraction is used to describe all gas from a raw coal mixing with the oxidizing gas. Coal gas is a
 163 mixture of a large number of different species with a wide variety of composition and energy
 164 properties. The fuel mixture fraction (as shown generally in Equation 1) of a one-mixture
 165 fraction system is shown in Equation 4:

166

$$167 \quad f = \frac{M_1}{M_0 + M_1} \quad (4)$$

168

169 where f is the fuel mixture fraction (coal or any number of coal surrogate gases), M_0 is the mass
 170 of the oxidizer stream, and M_1 is the mass of the fuel stream. The mass fraction of any element in
 171 the system can be found using the elemental mass fractions (shown generally in Equation 2), as
 172 shown in the Equations 5 to 9 (in order of CHONS):

173

$$174 \quad Z_C^1 = W_C \left(\frac{a_{C,1}f}{W_1} + \frac{a_{C,0}(1-f)}{W_0} \right) \quad (5)$$

$$175 \quad Z_H^1 = W_H \left(\frac{a_{H,1}f}{W_1} + \frac{a_{H,0}(1-f)}{W_0} \right) \quad (6)$$

$$176 \quad Z_O^1 = W_O \left(\frac{a_{O,1}f}{W_1} + \frac{a_{O,0}(1-f)}{W_0} \right) \quad (7)$$

$$177 \quad Z_N^1 = W_N \left(\frac{a_{N,1}f}{W_1} + \frac{a_{N,0}(1-f)}{W_0} \right) \quad (8)$$

$$178 \quad Z_S^1 = W_S \left(\frac{a_{S,1}f}{W_1} + \frac{a_{S,0}(1-f)}{W_0} \right) \quad (9)$$

179

180 where Z_C^1 is the elemental mixture fraction of carbon in a one-mixture fraction system (denoted
 181 by superscript 1), W_C is the molecular weight of carbon, $a_{C,1}$ is the number of atoms of carbon in

182 the fuel stream (estimated using the mole fraction of carbon in the case of coal-based fuels), f is
183 the fuel mixture fraction (mass fraction of fuel), and W_1 is the molecular weight of the fuel
184 stream. Similar variables are used for each element, with a subscript of zero used to denote
185 properties of the oxidizer stream added to the elemental mixture fractions of oxygen and nitrogen
186 (the two elements that are in the oxidizer stream). These are generalized equations, but for a
187 typical coal combustion application, the elemental mixture fractions would be simpler. Air does
188 not typically include any carbon, hydrogen, or sulfur, so the $a_{i,0}$ values (e.g., $a_{C,0}$) for each of
189 those elements would be zero. These values would be different if different oxidizer gases were
190 used (e.g., oxy-fuel combustion). Because there is only one fuel stream, only one equivalence
191 ratio (shown generally in Equation 3) is needed to describe this system. The one-mixture fraction
192 equivalence ratio is shown in Equation 10.

193

$$194 \quad \phi = \frac{\frac{n_1}{n_0}}{\left(\frac{n_1}{n_0}\right)_{stoich}} \quad (10)$$

195

196 where subscripts of zero and one are again used to denote the moles of the oxidizer and fuel
197 streams, respectively.

198

199 1.1.2 Two-Mixture Fraction Comparison

200 A two-mixture fraction system is a little more complex than a one-mixture fraction
201 system. Instead of only using a single fuel stream, a two-mixture fraction system includes two
202 fuel streams. While a two-mixture fraction method does not usually provide any benefit for
203 simple hydrocarbon combustion reactions, these methods can benefit coal combustion reactions
204 since pyrolysis and char combustion occur on different time scales. Coal reaction products can

205 therefore be treated as volatile gases (named the total volatiles) and the solid that remains after
 206 pyrolysis (named the char). These two general categories of pyrolysis products comprise the two
 207 fuel streams of a two-mixture fraction system. These two fuel mixture fractions are described in
 208 Equations 11 and 12.

$$210 \quad f_1 = \frac{M_1}{M_0 + M_1 + M_2} \quad (11)$$

$$211 \quad f_2 = \frac{M_2}{M_0 + M_1 + M_2} \quad (12)$$

212
 213 where the oxidizer stream is again denoted with a zero subscript, and subscripts of 1 and 2 are
 214 used to denote the mass and mixture fractions of the char and volatiles streams, respectively.
 215 Like all mixture fractions here, f_1 and f_2 sum to one minus the oxidizer mixture fraction (f_0).
 216 The mass fractions of the major elements in the combustion system can again be calculated using
 217 the elemental mixture fractions in Equations 13 to 17.

$$219 \quad Z_C^2 = W_C \left(\frac{a_{C,1}f_1}{W_1} + \frac{a_{C,2}f_2}{W_2} + \frac{a_{C,0}(1-f_1-f_2)}{W_0} \right) \quad (13)$$

$$220 \quad Z_H^2 = W_H \left(\frac{a_{H,1}f_1}{W_1} + \frac{a_{H,2}f_2}{W_2} + \frac{a_{H,0}(1-f_1-f_2)}{W_0} \right) \quad (14)$$

$$221 \quad Z_O^2 = W_O \left(\frac{a_{O,1}f_1}{W_1} + \frac{a_{O,2}f_2}{W_2} + \frac{a_{O,0}(1-f_1-f_2)}{W_0} \right) \quad (15)$$

$$222 \quad Z_N^2 = W_N \left(\frac{a_{N,1}f_1}{W_1} + \frac{a_{N,2}f_2}{W_2} + \frac{a_{N,0}(1-f_1-f_2)}{W_0} \right) \quad (16)$$

$$223 \quad Z_S^2 = W_S \left(\frac{a_{S,1}f_1}{W_1} + \frac{a_{S,2}f_2}{W_2} + \frac{a_{S,0}(1-f_1-f_2)}{W_0} \right) \quad (17)$$

224

225 where Z_c^2 is the elemental mixture fraction of carbon in a two-mixture fraction system (note, the
226 superscript 2 does not indicate a squared value, only that the elemental mixture fraction describes
227 that of a two-mixture fraction system), and the other variables are extensions of those previously
228 described. Like the one-mixture fraction comparison, the five elemental mixture fractions sum to
229 one and would simplify based on the oxidizer gases used. In addition to the two fuel mixture
230 fractions, there are also two independent equivalence ratios that can be used to determine the
231 amount of oxidizer in the system, shown in Equations 18 and 19.

232

$$233 \quad \phi_1 = \frac{\frac{n_1}{n_0}}{\left(\frac{n_1}{n_0}\right)_{stoich}} \quad (18)$$

$$234 \quad \phi_2 = \frac{\frac{n_2}{n_0}}{\left(\frac{n_2}{n_0}\right)_{stoich}} \quad (19)$$

235

236 with subscripts of 0, 1, and 2 describing the moles of oxidizer, char, and volatiles, respectively.

237

238 1.1.3 Three-Mixture Fraction Comparison

239 The three-mixture fraction comparison adds a third fuel mixture fraction. Like the two-
240 mixture fraction comparison, three mixture fractions generally are not useful in most simple
241 hydrocarbon combustion applications. Coal combustion systems, however, might benefit from
242 three mixture fractions. When coal reacts, the total volatiles can be further divided into two
243 additional categories: tars (volatile gases that condense to a sticky liquid at room temperature)
244 and light gases (volatile gases that remain as gas at room temperature). This leads to three total
245 fuel streams—one for the char, one for the tar, and a third for the light gases. The three fuel
246 streams are described by three fuel mixture fractions, shown in Equations 20 to 22:

$$247 \quad f_1 = \frac{M_1}{M_0+M_1+M_2+M_3} \quad (20)$$

$$248 \quad f_2 = \frac{M_2}{M_0+M_1+M_2+M_3} \quad (21)$$

$$249 \quad f_3 = \frac{M_3}{M_0+M_1+M_2+M_3} \quad (22)$$

250

251 where a subscript of zero again describes the oxidizer stream values, a subscript of 1 corresponds
 252 to the char stream, a subscript of 2 details the tar stream, and a subscript of 3 indicates the light
 253 gas stream. All three fuel mixture fractions and the oxidizer mixture fraction (f_0) again sum to 1.

254 The mass fractions of each element in the mixture can again be found by calculating the
 255 elemental mixture fractions, in Equations 23 to 27:

256

$$257 \quad Z_C^3 = W_C \left(\frac{a_{C,1}f_1}{W_1} + \frac{a_{C,2}f_2}{W_2} + \frac{a_{C,3}f_3}{W_3} + \frac{a_{C,0}(1-f_1-f_2-f_3)}{W_0} \right) \quad (23)$$

$$258 \quad Z_H^3 = W_H \left(\frac{a_{H,1}f_1}{W_1} + \frac{a_{H,2}f_2}{W_2} + \frac{a_{H,3}f_3}{W_3} + \frac{a_{H,0}(1-f_1-f_2-f_3)}{W_0} \right) \quad (24)$$

$$259 \quad Z_O^3 = W_O \left(\frac{a_{O,1}f_1}{W_1} + \frac{a_{O,2}f_2}{W_2} + \frac{a_{O,3}f_3}{W_3} + \frac{a_{O,0}(1-f_1-f_2-f_3)}{W_0} \right) \quad (25)$$

$$260 \quad Z_N^3 = W_N \left(\frac{a_{N,1}f_1}{W_1} + \frac{a_{N,2}f_2}{W_2} + \frac{a_{N,3}f_3}{W_3} + \frac{a_{N,0}(1-f_1-f_2-f_3)}{W_0} \right) \quad (26)$$

$$261 \quad Z_S^3 = W_S \left(\frac{a_{S,1}f_1}{W_1} + \frac{a_{S,2}f_2}{W_2} + \frac{a_{S,3}f_3}{W_3} + \frac{a_{S,0}(1-f_1-f_2-f_3)}{W_0} \right) \quad (27)$$

262

263 where Z_C^3 describes the elemental mixture fraction of carbon in a three-mixture fraction system
 264 (denoted by a superscript 3, note that this does not indicate a cubed value), and the remaining
 265 variables are likewise extensions of those described in the one- and two-mixture fraction
 266 comparisons. Additionally, there are three independent equivalence ratios in a three-mixture
 267 fraction system:

268
$$\phi_1 = \frac{\frac{n_1}{n_0}}{\left(\frac{n_1}{n_0}\right)_{stoich}} \quad (28)$$

269
$$\phi_2 = \frac{\frac{n_2}{n_0}}{\left(\frac{n_2}{n_0}\right)_{stoich}} \quad (29)$$

270
$$\phi_3 = \frac{\frac{n_3}{n_0}}{\left(\frac{n_3}{n_0}\right)_{stoich}} \quad (30)$$

271

272 1.2 Equilibrium Programs

273 Two widely-used chemical equilibrium programs for combustion systems are used here:
274 the stand-alone version of the NASA-CEA program [62, 63] and the Python interface of Cantera
275 [64]. Both programs calculate equilibrium states based on an input of initial conditions, including
276 chemical and thermodynamic properties of reactants, relying primarily on Gibbs free energy
277 minimization. Both programs also use similar thermodynamic property models with polynomial
278 representations of heat capacity, enthalpy, and entropy states of chemical species. Additionally,
279 both programs calculate an adiabatic equilibrium temperature, which can be used as a
280 comparison between fuels, however, real combustion systems are never truly adiabatic. Large-
281 scale simulations would take this into account by including a heat loss term. The NASA code is
282 not set up to deal with any heat loss in a real system, however, Cantera is flexible enough that it
283 could be used in conjunction with stand-alone research codes that adequately address heat loss
284 concerns.

285 The NASA-CEA program has been used to calculate equilibrium states of all types of
286 fuels, including solid fuels. In order to equilibrate coal systems, some novel subroutines were
287 developed to create Cantera mechanism files for coal-like fuels. These subroutines use the
288 elemental composition, enthalpy of formation, and the reference temperature at which the

289 enthalpy of formation is calculated. These inputs were used to develop coefficients for the
290 polynomial thermodynamic property models. Finally, the thermodynamic coefficients and other
291 relevant information were included in a mechanism file. These subroutines can be applied to
292 calculate the Cantera mechanism files for any coal-based fuels. Some instabilities were observed
293 in Cantera equilibrium calculations when just the coal mechanism files were used. To eliminate
294 these instabilities, the Cantera mechanism for solid carbon (graphite) had to be introduced along
295 with mechanism files for the unreacted coal (or other solid fuel) and combustion/background
296 gases. Additional mechanism files were created for the coal surrogate and combustion gases.

297 A second subroutine was created around the Cantera solver to iterate over the range of
298 equivalence ratios for each fuel stream. The mixture fractions were calculated at each
299 equivalence ratio and for each equilibrium state. The additional subroutine made the Cantera
300 calculations more convenient, especially for the two- and three-mixture fraction comparisons.

301

302 1.3 Experimental Data

303 Experimental data were gathered from several sources based on the following criteria:
304 (a) reported elemental compositions must be reported on a dry, ash-free (DAF) basis or enough
305 information provided to calculate the elemental compositions on a DAF basis; (b) heating values
306 (enthalpies of combustion) must be reported on a DAF basis, or enough information provided to
307 calculate them on a DAF basis; (c) comparisons using two mixture fractions must have average
308 elemental compositions and heating values for both char and either tar or total volatiles; and (d)
309 data for the three-mixture fraction comparison must have enough information to calculate
310 elemental compositions and heating values for coal char, tar, and light gas. A review of a large
311 set of experimental heating values was reported by Richards, et al. [65], and all coal-based fuels

312 are taken from that data set. Some common coal surrogate gases are also used here. All fuels
 313 used, along with their respective sources, are described in Table 1.

314

315 Table 1. Fuels used in this analysis

Fuel No.	Fuel Name	Source	Coal Rank
F-1	Graphite	Perry's Handbook [66]	N/A
F-2	Benzene	Perry's Handbook [66]	N/A
F-3	Methane	Perry's Handbook [66]	N/A
F-4	Ethane	Perry's Handbook [66]	N/A
F-5	Ethylene	Perry's Handbook [66]	N/A
F-6	Pittsburgh #8 (Pitt 8) Coal	Proscia et al. [67]	HVA
F-7	Lower Kittanning Coal	Proscia et al. [67]	LVB
F-8	Millmerran Coal	Edwards et al. [68]	Subbituminous
F-9	Liddell Coal	Edwards et al. [68]	Bituminous
F-10	Mammoth Seam Coal	Miller [69]	Anthracite
F-11	Beulah Zap Coal	Miller [69]	Lignite
F-12	Buck Mountain Coal	Miller [69]	Anthracite
F-13	#8 Leader Seam Coal	Miller [69]	Anthracite
F-14	#8 Seam Coal	Miller [69]	Semi-anthracite
F-15	Gunnison Coal	Miller [69]	Semi-anthracite
F-16	L. Spadra Seam Coal	Miller [69]	Semi-anthracite

316

317 All 16 fuels were used in the one-mixture fraction comparison, with fewer in the two-
318 mixture fraction comparison, and only one used (F-6, Pitt 8) in the three-mixture fraction
319 comparison. The fuel cases for each mixture fraction analysis are described in the Results and
320 Discussion section.

321 While the elemental compositions and heating values of most of the fuels listed in Table
322 1 can be found in Richards, et al [65], a brief summary of some of the compositional differences
323 between the fuels used here is warranted. Table 2 shows the DAF wt% of carbon, hydrogen,
324 oxygen, nitrogen, and sulfur for a few of the coal-based fuels. This table is to illustrate some of
325 the compositional differences that occur between different coal-based fuels, including different
326 coals, chars, and tars.

327

328 Table 2. Summary of Elemental Composition of Some Coal-Based Fuels

Fuel No.	Fuel Name	C	H	O	N	S
F-6	Pitt 8 Coal	82.36	5.51	8.56	1.65	1.92
F-6a	Pitt 8 Char	83.01	5.24	8.23	1.68	1.84
F-6b	Pitt 8 Tar	85.02	6.40	5.68	1.63	1.27
F-6c	Pitt 8 Volatiles	78.65	7.05	10.47	1.48	2.35
F-6d	Pitt 8 Light Gases	49.18	10.06	32.66	0.78	7.32
F-8	Millmerran Coal	78.40	6.70	13.10	1.20	0.60
F-11	Beulah Zap Coal	73.10	4.50	20.60	1.00	0.80
F-12	Buck Mountain Coal	90.80	2.60	5.20	0.80	0.60

329

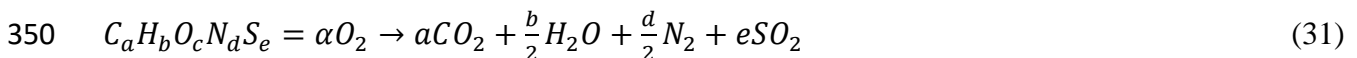
330 Different coals and coal-based fuels have very different compositions, and this affects the
331 compositions of combustion gases at equilibrium. The coal surrogate gases vary even more, with
332 none having any oxygen, nitrogen, or sulfur as part of the fuel.

333

334 2. Approach

335 The one-mixture fraction analysis used both the Cantera and NASA-CEA programs. The
336 NASA code is trusted in the coal community for evaluating equilibrium states of coal-based
337 fuels, so it is used to evaluate Cantera as a viable alternative for calculating equilibrium states.
338 Both equilibrium programs require similar inputs depending on if the fuel is included in the
339 program data or is user-defined. Simple fuels are included in the NASA-CEA code and are
340 directly available to Cantera via provided mechanism files; these include all five of the surrogate
341 gases: graphite (carbon solid), benzene, methane, ethane, and ethylene. These simple fuels
342 include all the composition and thermochemical properties required for the respective
343 equilibrium programs. User-defined compounds need two main parameters for both programs:
344 elemental composition (CHONS, on either a mass or mole basis) and an enthalpy (or heat) of
345 formation. For most coal-based fuels, elemental compositions and heats of combustion (also
346 called heating values) are reported for each fuel. Heats of formation can then be calculated for
347 each fuel by converting from heating values by assuming complete combustion, as shown in
348 Equations 31-33.

349



$$351 \quad \alpha = a + \frac{b}{2} - \frac{c}{2} + e \quad (32)$$

$$352 \quad \Delta H_r = \left(\sum_{i=1}^n \nu_i \Delta H_{i,f} \right)_{prod} - \left(\sum_{i=1}^n \nu_i \Delta H_{i,f} \right)_{react} \quad (33)$$

353 where a , b , c , d , and e are average molar compositions of the fuel, α is the stoichiometric
354 amount of oxygen needed for complete combustion, ΔH_r is the enthalpy of reaction (also called
355 heating value), v_i is a generic stoichiometric coefficient of compound i , $\Delta H_{i,f}$ is the enthalpy of
356 formation of compound i , subscripts *prod* and *react* designate properties of the products (CO₂,
357 H₂O, etc.) and reactants (fuel and O₂), respectively.

358 In addition to supplying the equilibrium programs with the properties of each fuel, the
359 mass or moles of each stream is necessary to complete equilibrium calculations. The NASA-
360 CEA code allows the input of a list of equivalence ratios, which makes calculating the
361 equilibrium states using NASA-CEA more convenient. Cantera needs the total amount of
362 oxidizer, which can be calculated either using the equivalence ratio or fuel mixture fraction.
363 Equilibrium calculations using the one-mixture fraction approach were performed for a range of
364 equivalence ratios from 0.1 (fuel-lean) to 3 (fuel-rich), with a greater number of points around
365 the stoichiometric point (equivalence ratio of 1). The one-mixture fraction comparison used this
366 equivalence ratio range to calculate the total air necessary to calculate equilibrium states for each
367 fuel.

368 The procedure became more complicated with the two- and three-mixture fraction
369 analyses. Because the analysis used a range of equivalence ratios rather than mixture fractions to
370 determine the appropriate amounts of air for each fuel, a separate “mixing condition” was needed
371 to determine the ratio of the fuel streams and fully define the system. If the fuel mixture fraction
372 was used instead of the equivalence ratio, this fuel mixing condition would not be necessary.
373 This fuel mixing condition is simply how much of each fuel stream is present prior to mixing
374 with the oxidizer stream. The fuel mixing condition is allowed to vary between 0 and 1, with the
375 sum of the fuel stream percentages not exceeding one.

376 In a two-mixture fraction system, there are two extremes in defining the fuel streams: (a)
377 pure fuel 1 (char) with no fuel 2 (volatiles) and (b) pure fuel 2 (volatiles) with no fuel 1 (char).
378 The first extreme would be defined as a fuel mixing condition of 0, which means that the fuel in
379 the fuel mixture would only consist of fuel 1 (char) and the other extreme would correspond to a
380 fuel mixing condition of 1, or 100 percent volatiles in the fuel. To span this fuel mixing space,
381 five main fuel mixing cases were created with fuel ratios corresponding to 0, 25, 50, 75, or 100%
382 volatiles (called Y_{vol}). This means that at 0% volatiles, the fuel is 100% char and vice-versa.
383 Additionally, the reported pyrolysis yield for each fuel was used as a sixth fuel mixing ratio
384 (between 10 and 60% depending on the fuel and pyrolysis conditions). Air was added to each
385 fuel stream to vary the equivalence ratio independently between 0.1 and 3, with the total amount
386 of oxidizer being a combination of the two equivalence ratios. This procedure was followed for
387 all six fuel mixing ratios.

388 The three-mixture fraction comparison was slightly more complex, but a similar
389 procedure to the two-mixture fraction comparison was followed to span the three-mixture
390 fraction fuel mixing space. The three fuel streams (char, tar, and light gas) were allowed to vary
391 among the same six values as the two-mixture fraction comparison (i.e., 0, 25, 50, 75, and 100%,
392 and the reported pyrolysis yield of char, tar, and light gas). There are, however, two constraints
393 placed on these fractions: (a) the three fuel fractions sum to one and (b) all fuel fractions must be
394 between 0 and 1 (no negative numbers). Air was added to each fuel stream to vary the
395 equivalence ratio between 0.1 and 3 for each fuel stream independently. The total amount of air
396 is calculated for each fuel mixing condition and equivalence ratio. This procedure generated six
397 total fuel mixing conditions for each two-mixture fraction comparison and 21 for each three-
398 mixture fraction comparison.

399 Using the elemental compositions and enthalpies of formation for each fuel, and setting
400 the pressure to one atmosphere, the equilibrium programs calculate the compositions of a large
401 number of combustion gases for each equilibrium state. This analysis used six key variables from
402 each equilibrium state to compare between fuels and mixture fraction methods: temperature and
403 the mole fractions of O₂, CO₂, CO, H₂O, and graphite (i.e., solid carbon). This equilibrium
404 temperature is an adiabatic temperature and does not account for any heat loss that might be
405 observed in any real-world combustion applications.

406 Several statistical parameters [40, 65] were used to quantify error between the
407 equilibrium states calculated by both NASA-CEA and Cantera, however, only one is presented
408 here: the root-mean-square error (RMSE), found in Equation 7. The remainder of the statistical
409 parameters and their results are discussed in Richards [70].

410

$$411 \quad RMSE = \left(\frac{1}{N} \sum_{n=1}^N (\hat{y}_n - y_n)^2 \right)^{\frac{1}{2}} \quad (34)$$

412

413 where y_n is the “true” value, \hat{y}_n is the “questioned value”, N is the total number of points in the
414 comparison set. The equation for RMSE is a statistical parameter where experimentally observed
415 and model prediction values are generally used in place of the “true” and “questioned” values,
416 respectively. Both NASA and Cantera give predicted values for equilibrium states, so in order to
417 adequately compare NASA to Cantera, in this analysis the NASA values are considered to be the
418 “true” values and the Cantera are the “questioned” values. This distinction is not as important
419 with the RMSE, however, it does make a slight difference in other statistical parameters (see
420 Richards [70]).

421

422 3. Results and Discussion

423 The results are discussed based on the number of fuel mixture fractions considered,
424 starting with the one-mixture fraction comparison, followed by the two-mixture fraction
425 comparison, and finally the three-mixture fraction approach compared to the one- and two-
426 mixture fraction approaches. Each section consists of a description of the fuels used for each
427 case. After the results are presented and discussed, a final section summarizes the key findings
428 and discusses application to physical processes. Additional details and discussion are found in
429 Richards [70].

430

431 3.1 One Mixture Fraction

432 The one-mixture fraction approach breaks the components into one fuel stream (e.g.,
433 coal) and one oxidizer stream (air). These two streams are mixed in different proportions
434 according to the equivalence ratio. The fuels used in the one-mixture fraction analysis are listed
435 in Table 3.

436

437 Table 3. One-Mixture Fraction Fuel Cases

Case Number	Fuel Number	Fuel Name	Coal Rank
1-1	F-1	Graphite	N/A
1-2	F-2	Benzene	N/A
1-3	F-3	Methane	N/A
1-4	F-4	Ethane	N/A
1-5	F-5	Ethylene	N/A
1-6	F-6	Pitt 8 Coal	HVA

1-7	F-6a	Pitt 8 Char	HVA, char
1-8	F-6b	Pitt 8 Tar	HVA, tar
1-9	F-7	Lower Kittanning Coal	LVB
1-10	F-8	Millmerran Coal	Subbituminous
1-11	F-9	Liddell Coal	Bituminous
1-12	F-10	Mammoth Coal	Anthracite
1-13	F-11	Beulah Zap Coal	Lignite
1-14	F-12	Buck Mountain Coal	Anthracite
1-15	F-13	#8 Leader Seam Coal	Anthracite
1-16	F-14	#8 Seam Coal	Semi-anthracite
1-17	F-15	Gunnison Coal	Semi-anthracite
1-18	F-16	L. Spadra Seam Coal	Semi-anthracite

438

439 Two main questions were addressed in the one-mixture fraction comparison: (1) how
440 closely do the results from the NASA-CEA and Cantera programs compare to each other; and (2)
441 what is the difference between the various coal surrogates and several different coal types? The
442 comparison between the NASA and Cantera programs is briefly discussed first followed by a
443 comparison of the different cases from Table 3.

444

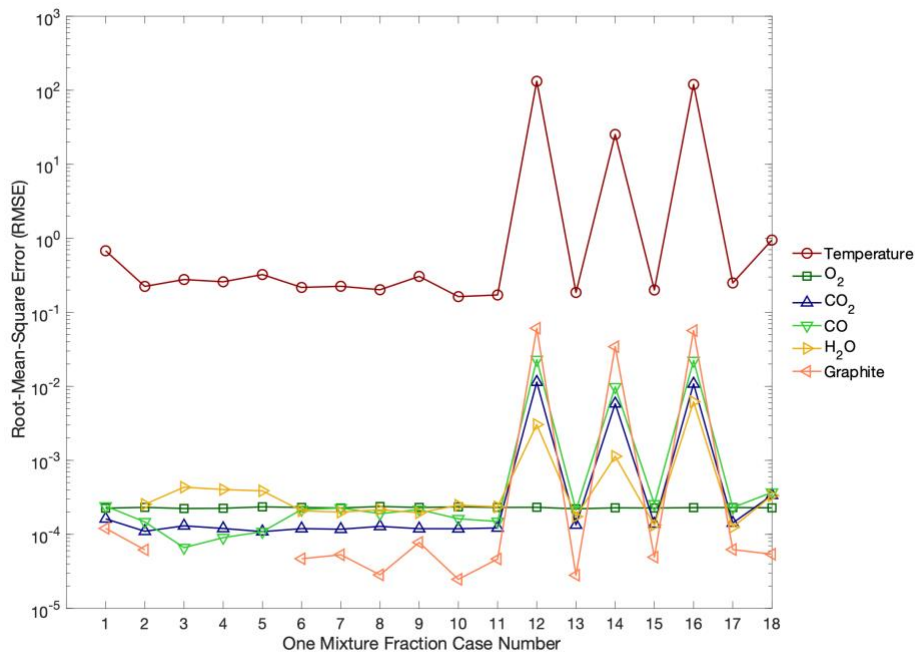
445 3.1.1 NASA-CEA vs Cantera

446 The NASA equilibrium code has been widely used in the coal community over the years
447 and is considered a standard in coal equilibrium calculations, but Cantera offers flexibility that

448 makes its integration into existing codes beneficial for large-scale simulations. This study was
449 therefore performed to determine how closely the Cantera results align with the NASA results.

450 Both Cantera and NASA-CEA were used to calculate equilibrium states for all fuel cases
451 listed in Table 3, comparing temperature and the mole fractions of O₂, CO₂, CO, H₂O, and
452 graphite (solid carbon). Figure 1 shows the root-mean-square error (RMSE) between the NASA
453 and Cantera results (additional error analysis is given by Richards [70]). Note that this is the
454 RMSE over the entire range of equivalence ratio ranging from 0.1 to 3.0.

455



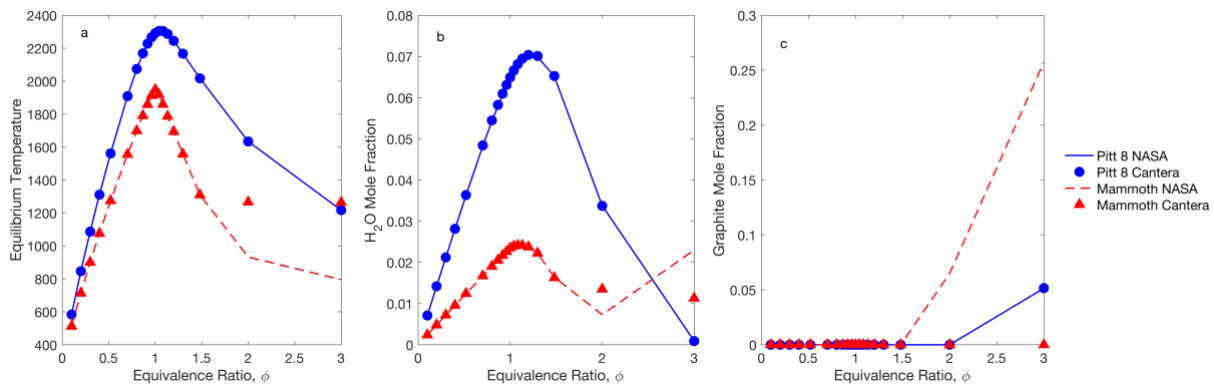
456

457 Figure 1. Root-mean-square error between NASA and Cantera Results

458

459 For the most part, there is good agreement between the NASA and Cantera results,
460 however, a few of the highest rank coals (anthracites and semi-anthracites), the RMSE is quite
461 high. From the statistical summary in Figure 1, it is unclear if those spikes in RMSE are a result

462 of the deviation of a single point or multiple points in the equivalence ratio range. Results from
 463 NASA and Cantera were plotted against each other for each fuel over the range of equivalence
 464 ratios to determine where the deviations occur. Figure 2 shows an example of good agreement
 465 between NASA and Cantera (Pitt 8 coal, case number 1-6) and poor agreement (Mammoth seam
 466 coal, case number 1-12) for equilibrium temperature and the mole fractions of H₂O and graphite.
 467 These parameters help to illustrate the reason for the deviation between NASA and Cantera and
 468 in which conditions the deviations occur. The lines are from NASA-CEA and the points are from
 469 Cantera. More detail on these equilibrium comparisons are found in Richards [70].
 470



471
 472 Figure 2. Comparison of NASA (lines) to Cantera (markers) results: (a) Temperature, (b) H₂O
 473 mole fraction, and (c) graphite mole fraction.

474
 475 The statistical results shown in Figure 1 give an overall picture as to which fuels exhibit
 476 poor agreement. However, the poor agreement occurs only in fuel-rich conditions where $\phi >$
 477 1.5, as illustrated in Figure 2. NASA converts all unreacted fuel to graphite (indicated by the
 478 higher graphite mole fractions at higher equivalence ratios) whereas Cantera tends to leave some
 479 portion of the higher rank coals as unreacted fuel with the same composition and energy

480 properties. This is likely because of the way the Cantera equilibrium solver was set up to reduce
481 numerical instabilities, which included separate mechanisms for the combustion gases, graphite,
482 and the solid fuel. Because Cantera leaves some of the material as unreacted coal instead of
483 converting it to graphite, the amount of graphite present at the highest equivalence ratios as well
484 as the amounts of the other equilibrium products are decreased. The unreacted coal “species”
485 contains higher amounts of hydrogen, oxygen, and other atoms than graphite. The disparity
486 between NASA and Cantera only seems to occur with some higher rank coals. Most modern
487 applications of coal combustion will fall outside of these extreme conditions when looking at the
488 system as a whole, save for gasification or other specialized processes. However, there will
489 likely be regions that are locally very fuel rich, especially in regions of coal pyrolysis where
490 there are typically low concentrations of oxidizing gases close to the coal. Even with these few
491 cases of deviation between NASA and Cantera, there is a low instance of disparity overall, which
492 means Cantera can be used to adequately calculate the equilibrium states of multiple mixture
493 fraction systems (with the exception of a few of the highest rank coals), which is more difficult
494 to do with the stand-alone NASA code.

495

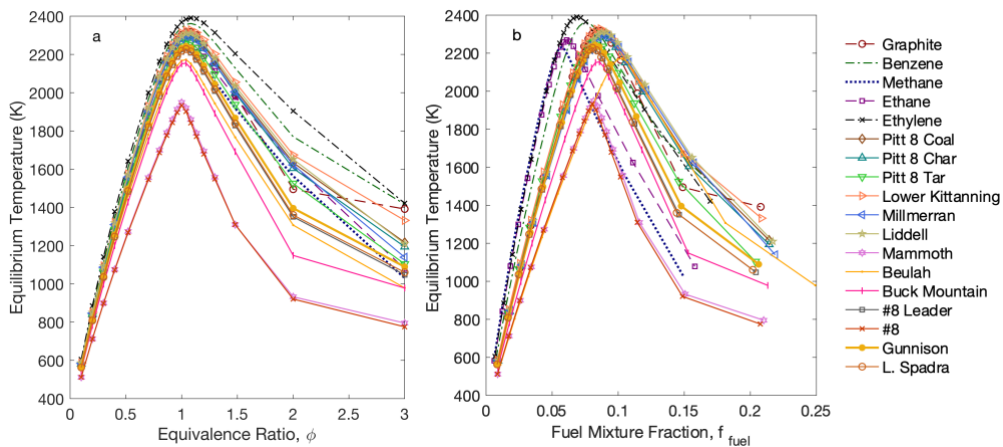
496 3.1.2 Full Fuel Comparison

497 The second question addressed by the one-mixture fraction comparison was to ascertain
498 the difference is between coal types and various coal surrogate gases. Coal surrogate gases such
499 as graphite, benzene, methane, ethylene and others have been used in the place of coal and coal-
500 based fuels [39]. While these can be used in a combination to reach the correct atomic ratios,
501 reaction and bond energies are harder to get correct, leading to inaccuracies in temperature and
502 other important equilibrium factors. Surrogate gas enthalpies can be adequately matched to coal-

503 specific values by changing the surrogate gas temperature to obtain the correct enthalpy,
 504 however, this can result in gas temperatures that are outside the bounds of the curve fits for the
 505 polynomial thermodynamic models, leading to inaccurate thermodynamic property predictions.
 506 The surrogates used in this analysis do not have adjusted enthalpies but are instead used as a
 507 direct substitute for coal-based fuels.

508 To address these inaccuracies and to get an idea of how much error might be introduced
 509 into a large-scale simulation by using these simplifying assumptions, all fuel cases from Table 3
 510 were directly compared against each other using equilibrium temperature and the mole fractions
 511 of O₂, CO₂, CO, H₂O, and graphite. Figure 3 shows the NASA results for equilibrium
 512 temperature, plotted using both the equivalence ratio and fuel mixture fraction. While this figure
 513 shows the results from the NASA code, the results from Cantera were virtually identical.

514



515

516 Figure 3. Equilibrium temperature calculations by the NASA-CEA code for all fuel cases: (a) vs.
 517 equivalence ratio and (b) vs. fuel mixture fraction.

518

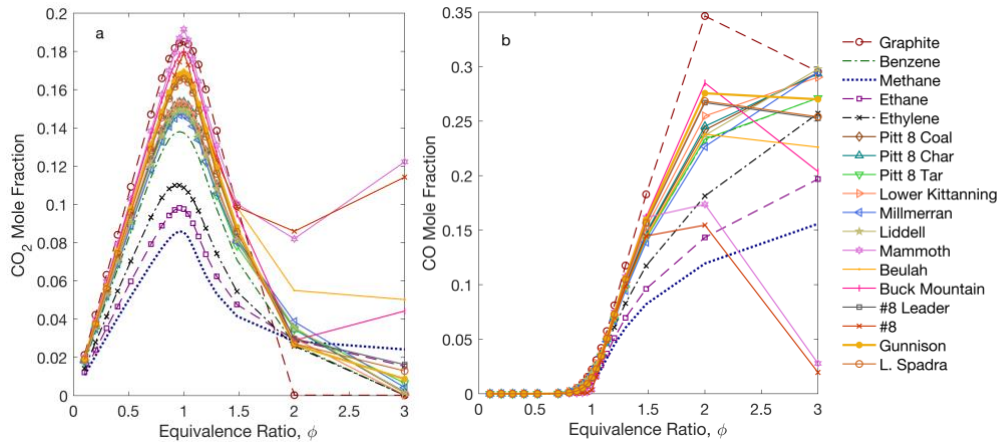
519 Equilibrium temperature is strongly influenced by fuel type used, especially at or above
 520 stoichiometric mixtures. The peak temperature usually occurs at a mixture just fuel rich of the

521 stoichiometric point and the difference is around 400 to 500 K between the highest temperature
522 fuel (ethylene) and the lowest temperature fuel (Mammoth seam anthracite). The temperature
523 difference is even greater in the fuel-rich conditions ($\phi > 1$). For most fuels, the difference is
524 minimal at very fuel lean conditions ($\phi \leq 0.5$), likely due to fuel being very dilute compared to
525 the oxidizer. Using the fuel mixture fraction complicates the comparison slightly, shifting the
526 peak temperatures to the left and right while keeping the peak temperature value the same. The
527 comparison vs. equivalence ratio therefore makes more sense and will be used in the remainder
528 of this paper. See Richards [70] for the remainder of the comparison vs. fuel mixture fraction.

529 The O₂ mole fraction comparisons did not show a lot of variability between fuels, and are
530 not shown here (see Richards [70]). This lack of variability in equilibrium O₂ between the fuels
531 is because most of the oxygen comes from the air with little to none from the fuel.

532 Figure 4 shows the CO₂ and CO mole fractions vs. equivalence ratio for the different
533 fuels. There is wide variability in the equilibrium CO₂ mole fraction depending on which fuel is
534 used, especially around stoichiometric conditions, which is where most industrial coal furnaces
535 operate. Many of the coals fall in the middle range of the peak CO₂ mole fraction, with the coal
536 surrogates lying toward both the top and bottom. The fraction of CO₂ would depend on the
537 carbon content of the original fuel, with fuels enriched in carbon (high rank coals and graphite)
538 generally having an increased CO₂ and CO content. The CO₂ content greatly influences the
539 overall heat transfer properties of a coal boiler since CO₂ blocks a lot of radiative heat transfer in
540 the gaseous environment. Generally, the ratio between CO₂ and CO are dependent on local gas
541 temperature and local oxygen content.

542



543

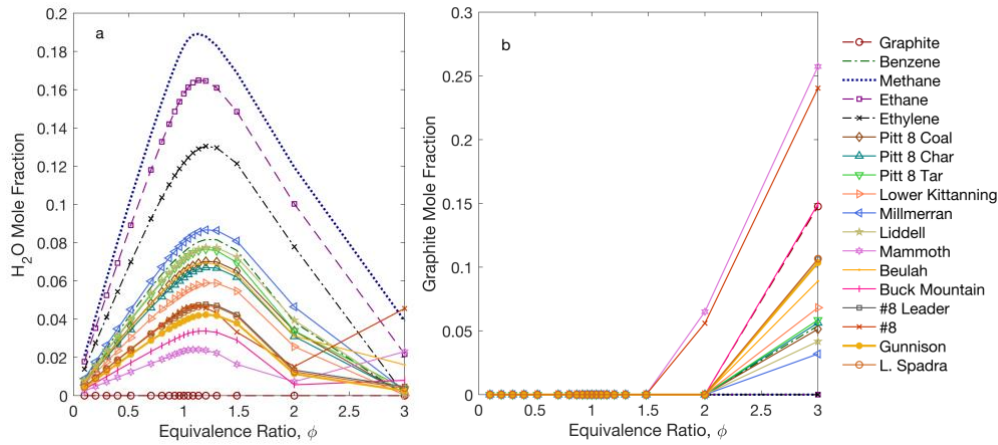
544 Figure 4. Equilibrium CO₂ and CO mole fraction calculations by the NASA-CEA code for all
 545 fuel cases: (a) CO₂ mole fraction vs. equivalence ratio and (b) CO mole fraction vs. equivalence
 546 ratio.

547

548 The CO mole fraction only becomes significant near stoichiometric conditions and
 549 moving into fuel rich conditions. This makes sense from a combustion perspective because there
 550 is less oxygen to completely oxidize the carbon in fuel rich conditions. The difference in
 551 calculated CO between fuels also increases at higher equivalence ratios. The decrease in CO (and
 552 increase in CO₂) at $\phi > 2$ is thought to be due to lower temperatures as well as the presence of
 553 graphite and unreacted fuel.

554 The H₂O and graphite mole fraction comparisons are shown in Figure 5. The H₂O mole
 555 fraction varies widely based on what fuel is used, from no H₂O with graphite to almost 20 mol%
 556 at the peak with methane. The amount will vary based solely on the amount of hydrogen in the
 557 original fuel since there is no additional hydrogen in the air. Also like the CO₂ mole fraction, the
 558 amount of water in a gaseous environment greatly influences the radiative heat transfer of the
 559 system. The equilibrium H₂O compositions for most coals again fall in the middle range of all
 560 fuels, with several surrogate gases lying to the extreme highs and lows of H₂O compositions.

561



562

563 Figure 5. Equilibrium H₂O and graphite mole fraction calculations by the NASA-CEA code for
564 all fuel cases: (a) H₂O mole fraction vs. equivalence ratio and (b) graphite mole fraction vs.
565 equivalence ratio.

566

567 Graphite only shows up in the most fuel rich conditions, acting as a surrogate for
568 “unreacted” fuel, char, and soot, with most fuel cases only showing graphite formation at
569 equivalence ratios of 2 and higher. Unreacted fuel is almost always a problem if it is present at
570 the end of combustion, especially when attempting to obtain the maximum amount of energy
571 from a fuel.

572 Using simple hydrocarbons as surrogate gases for coal and coal-based fuels (char, tar,
573 light gases, etc.) makes equilibrium calculations simpler and faster. However, using these
574 surrogates as a direct replacement for coal-based fuels introduces a large amount of variability,
575 especially at higher equivalence ratios (extreme fuel-rich conditions). This is particularly the
576 case for CO₂ and H₂O content, and to a lesser degree equilibrium temperature and CO content.
577 The majority of the inconsistencies between real coal values and surrogate gases could perhaps
578 be mitigated to a degree by dialing in the enthalpy of the surrogates by changing the reference

579 temperature at which the enthalpy is calculated, however, this could potentially lead to
580 improbable or even impossible temperatures. For these reasons, it is much better to use the actual
581 physical and chemical properties of the coal (when available) instead of simplifying it with a
582 surrogate gas.

583

584 3.2 Two Mixture Fractions

585 The two-mixture fraction comparison breaks the fuel stream into two separate streams,
586 one for mass originating in the char and one for the volatiles. Like the one-mixture fraction
587 comparison, there are two main questions addressed by the two-mixture fraction comparison: (1)
588 how close are the results using the reported tar properties rather than full volatiles (by combining
589 tar and light gas properties) and (2) can char and volatile surrogate gases be used in the place of
590 real char and volatiles properties? The first question is raised because some pyrolysis
591 experiments report only tar yields, compositions, and heating values but not light gas properties
592 with which to calculate values for the total volatiles. Table 4 shows the fuel cases tested in the
593 two-mixture fraction comparison. Note that Cantera was the only equilibrium program used in
594 this comparison as a matter of convenience.

595

596 Table 4. Two-Mixture Fraction Fuel Cases

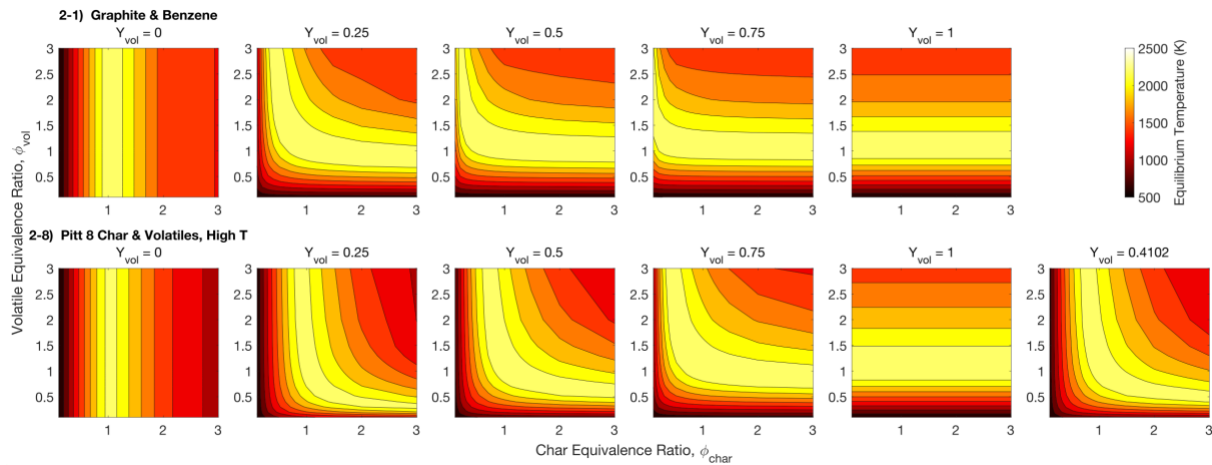
Case Number	Char	Volatiles	Reported Volatiles Yield
2-1	Graphite	Benzene	
2-2	Graphite	Methane	
2-3	Graphite	Ethane	

2-4	Graphite	Ethylene	
2-5	Pitt 8 Char (Low Temperature)	Pitt 8 TAR (Low Temperature)	0.1491
2-6	Pitt 8 Char (High Temperature)	Pitt 8 TAR (Low Temperature)	0.4102
2-7	Pitt 8 Char (Low Temperature)	Pitt 8 VOLATILES (Low Temperature)	0.1491
2-8	Pitt 8 Char (High Temperature)	Pitt 8 VOLATILES (High Temperature)	0.4102
2-9	Millmerran Char (High Temperature)	Millmerran TAR (High Temperature)	0.545
2-10	Millmerran Char (High Temperature)	Millmerran VOLATILES (High Temperature)	0.545
2-11	Millmerran Char (Low Temperature)	Millmerran TAR (Low Temperature)	0.313
2-12	Millmerran Char (Low Temperature)	Millmerran VOLATILES (Low Temperature)	0.313

597

598 To get the whole range of fuel mixing conditions, equilibrium states for each case were
599 calculated in the equivalence ratio range of 0.1 to 3 for each of 5 or 6 fuel mixing conditions
600 ranging from no volatiles to all volatiles ($Y_{vol} = 0, 25, 50, 75,$ and 100% of the fuel, and the
601 reported volatiles yield for each of the coals, which is an intermediate percentage between 10 and
602 60% of the DAF coal depending on pyrolysis conditions and coal type). The same variables of

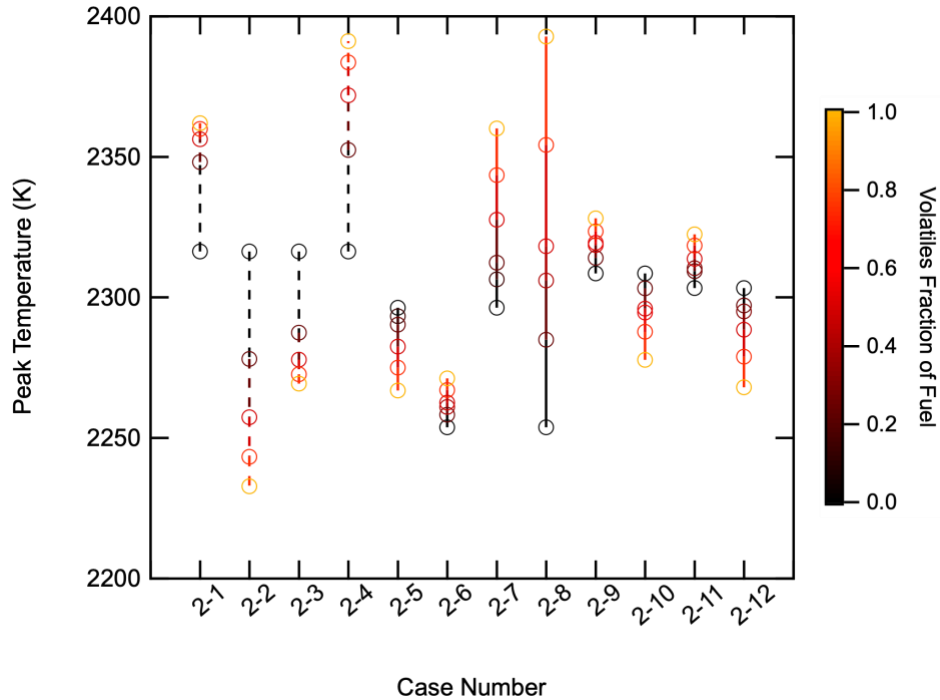
603 interest used in the one-mixture fraction comparison (temperature and the mole fractions of O₂,
 604 CO₂, CO, H₂O, and graphite) were compared for all cases listed in Table 4. To illustrate the
 605 temperature profiles for each fuel mixing condition, Figure 6 shows contour plots of the
 606 equilibrium temperature for two fuel cases (2-1, graphite and 2-8, Pitt 8 coal) for all fuel mixing
 607 conditions. The columns represent the percent of the fuel that was volatiles (vs. char) and the
 608 rows represent the given fuel considered.
 609



610
 611 Figure 6. Equilibrium temperature calculations by Cantera. Note that all plots have the same
 612 color range, as shown in the color bar in the upper right corner of the plot.

613
 614 Only two cases were used here to demonstrate the trends between cases and fuel mixing
 615 ratios. The plots for the other cases listed in Table 4 are found in Richards [70]. A given column
 616 in Fig. 6 shows a given fuel mixing condition for each type of fuel. The general shapes are
 617 similar, however, there are slight differences, especially with a higher volatiles mixture ($Y_{vol} \geq$
 618 75%). The difference in shape of the temperature curves is more pronounced between the coal-
 619 based fuels and the simpler surrogate gas fuels (not shown) but is less pronounced among the
 620 coal-based fuels, which is similar to the trends observed in the one-mixture fraction comparison.

621 In addition, the simpler fuels tend to have higher temperatures than the coal-based fuels,
 622 particularly around stoichiometric conditions. Figure 7 shows the peak temperature ranges of all
 623 fuel cases listed in Table 4. The circles for each case represent the peak equilibrium temperature
 624 for each value of Y_{vol} . For example, the peak temperatures in each of the subplots in Fig. 6 for
 625 Case 2-8 range from 2250 K for $Y_{vol} = 0$ to almost 2400 K for $Y_{vol} = 1$. These temperature
 626 differences, even small differences, can potentially impact the accuracy of large-scale
 627 simulations, especially in fuel-rich conditions where the difference is more pronounced.
 628



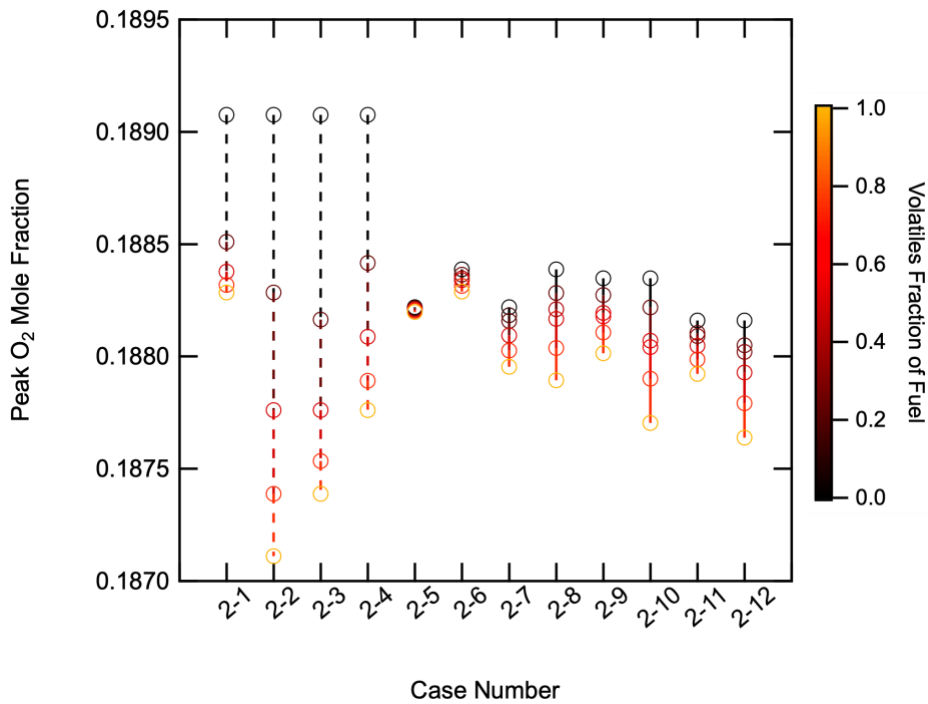
629
 630 Figure 7. Peak temperature range for all fuel cases. Dashed lines correspond to simple surrogate
 631 gases and the circles correspond to each fuel mixing condition. The color of the lines and dots
 632 depends on the fuel mixing ratio (Y_{vol} ranging from 0 to 100% volatiles).
 633

634 The first part of this analysis was to determine if measured tar properties could
635 reasonably be used as a surrogate for the total volatiles. There were four comparisons between
636 tar and total volatiles: 1) cases 2-5 and 2-7, 2) cases 2-6 and 2-8, 3) cases 2-9 and 2-10, and 4)
637 cases 2-11 and 2-12. Figure 7 shows that there is enough variability between the tar and volatiles
638 cases that it is not a good idea to use tar instead of total volatiles. For example, the temperature
639 range for case 2-6 spans less than 50 K while that for case 2-8 spans almost 150 K. Most of the
640 similarity in the calculated temperatures comes from the char-only fuel mixtures (black circles,
641 where $Y_{\text{vol}} = 0$). The extremes of char-only (black circles) and volatiles-only (yellow circles, Y_{vol}
642 = 1) fuel mixtures would fall directly in line with the one-mixture fraction results using char-only
643 or volatiles-only fuels. The one-mixture fraction results using the original coal properties of fuel
644 would likely be better compared to the two-mixture fraction results where the char and volatiles
645 are more evenly split (close to a Y_{vol} of 0.5) or the pyrolysis yield results (e.g., Y_{vol} of 0.4102).

646 The peak temperature depends highly on the fuels used in the two-mixture fraction
647 system. Some mixtures have larger ranges (2-8, which uses the measured properties of high
648 temperature Pitt 8 pyrolysis products), while many others have much smaller ranges. Some of the
649 surrogate gas cases are encompassed by the measured coal-based fuel cases, but others have no
650 overlap. There is a large variability in the range and limits of equilibrium temperature depending
651 highly on the fuels used. While these are peak adiabatic temperatures, which occur near $\phi_{\text{tot}} = 1$,
652 the discrepancies due to the fuel assumption would likely be very similar when used in real, non-
653 adiabatic systems.

654 The mole fractions of the key combustion-related species at equilibrium were also
655 compared in graphs like Figs. 6 and 7. Like the one-mixture fraction comparison, the O_2 mole
656 fraction does not differ much between fuel cases, so the contour plots are not shown here, but

657 can instead be found in Richards [70]. Because of the similarity of the equilibrium O₂ mole
 658 fractions between fuels, the shapes and ranges are all very similar. This is understandable
 659 because the majority of the oxygen content comes from the air, even with coal-based fuels that
 660 have oxygen bound in the organic matrix. The peak O₂ mole fractions for each fuel case and fuel
 661 mixing condition are shown in Figure 8.
 662



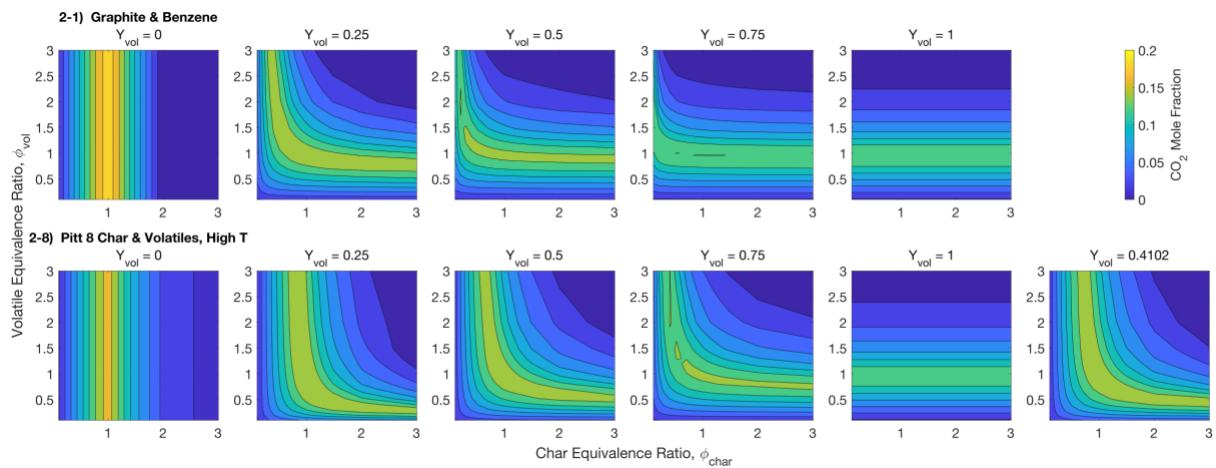
663
 664 Figure 8. Peak O₂ mole fraction range for all fuel cases. Dashed lines correspond to simple
 665 surrogate gases and the circles correspond to each fuel mixing condition. The color of the lines
 666 and dots depends on the fuel mixing ratio (Y_{vol} ranging from 0 to 100% volatiles).

667
 668 While Figure 8 seems to show a large variability in oxygen mole fraction, especially in
 669 the ranges between the simple coal gas surrogates (first four lines from the left) and the coal-
 670 based fuels, the axis range makes it seem larger. The peak O₂ mole fraction only ranges between

671 0.187 and 0.189. In addition, these peak O₂ mole fractions only occur at the lowest equivalence
 672 ratios, in extreme fuel-lean conditions. This shows that the equilibrium oxygen content is not
 673 highly dependent on the fuel used in equilibrium calculations, but more dependent on the fuel-to-
 674 oxidizer ratio used in the combustion process.

675 In the one-mixture fraction comparison, the CO₂ mole fraction varied highly depending
 676 on the fuel type used. A representative CO₂ mole fraction comparison of contour plots for the
 677 two-mixture fraction calculations are shown in Figure 9 (a complete series of plots are given in
 678 Richards [70]).

679



680

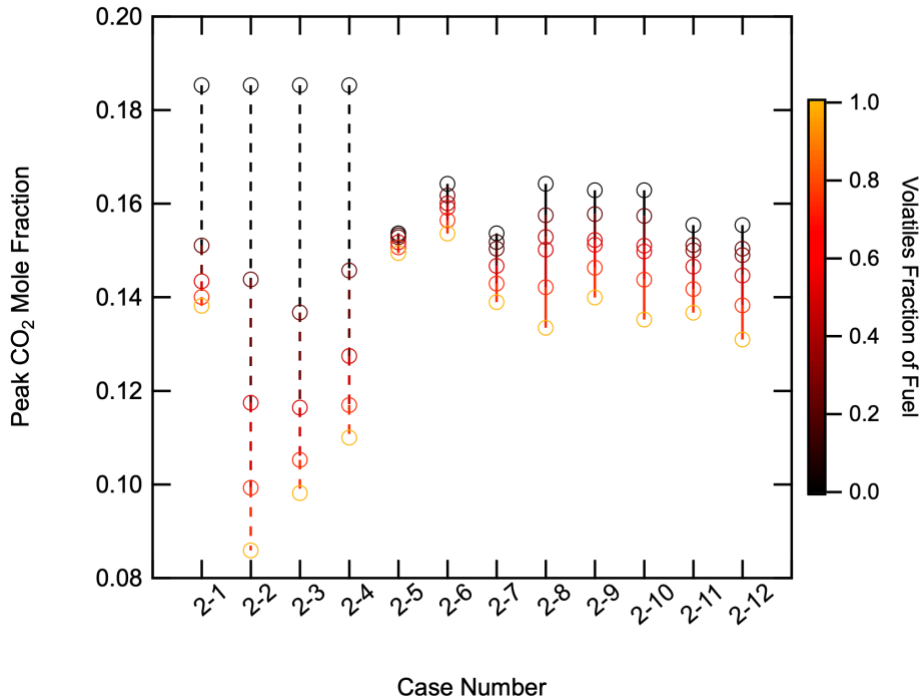
681 Figure 9. Equilibrium CO₂ mole fraction calculations by Cantera. Note that all plots have the
 682 same color range, as shown in the color bar in the upper right corner of the plot.

683

684 Like the one-mixture fraction comparison, CO₂ follows the same general trend as the
 685 temperature curves. However, like the temperature curves, the range of peak CO₂ mole fractions
 686 are very different for each fuel. There are major differences when comparing the coal-based
 687 mixtures to the coal surrogate gas mixtures. The four surrogate gas mixtures approximate the
 688 char as pure carbon (graphite) and the volatiles surrogate changes with each case. With more

689 char (graphite), the CO₂ fraction is much greater than for the coal-based mixtures at the same
 690 fuel mixing conditions, whereas when the volatiles are more favored in the mixture, the CO₂
 691 mole fraction is a lot lower than the coal-based mixtures. This is because of the large discrepancy
 692 in the carbon content of the coals and coal surrogate gases. Graphite is pure carbon (much higher
 693 in carbon than any of the coal-based fuels) and the other simple hydrocarbon fuels have a lower
 694 carbon fraction than the coal-based fuels. The ranges of peak CO₂ mole fractions for each fuel
 695 case are shown in Figure 10.

696



697

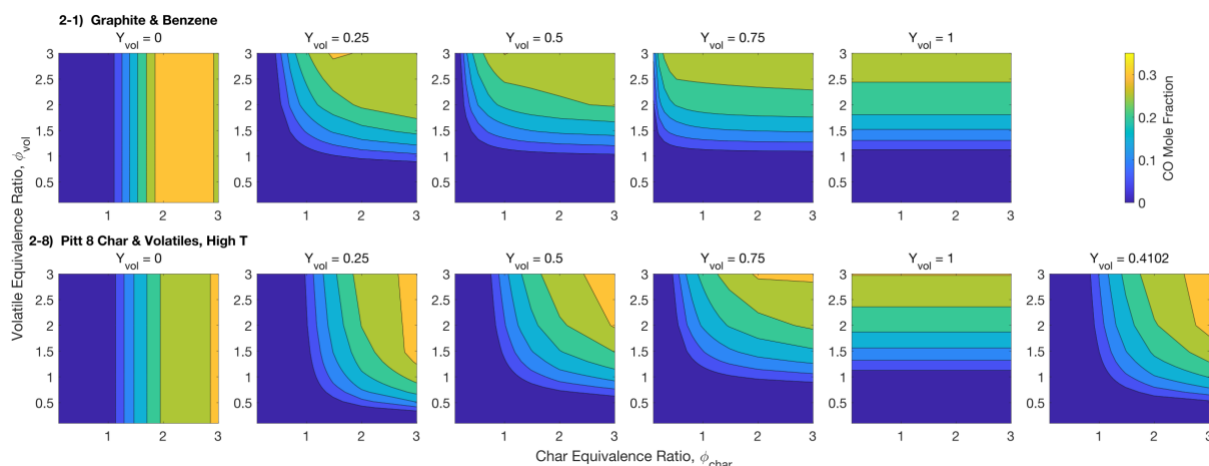
698 Figure 10. Peak CO₂ mole fraction range for all fuel cases. Dashed lines correspond to simple
 699 surrogate gases and the circles correspond to each fuel mixing condition. The color of the lines
 700 and dots depends on the fuel mixing ratio (Y_{vol} ranging from 0 to 100% volatiles).

701

702 The CO₂ mole fraction increases when more oxygen is present for complete combustion
 703 to occur. However, the peak CO₂ mole fraction differs from the O₂ mole fraction in the overall
 704 range of values. The peak CO₂ range is largest with the coal surrogate gases and smaller in the
 705 coal-based fuels, with a maximum range between 8 and 19%. Note that the coal-based fuels
 706 include some fuel-bound oxygen while the surrogates contain no additional oxygen.

707 The ratio between the CO₂ and CO mole fractions is dependent on both local temperature
 708 and O₂ content. Because of this, the CO/CO₂ ratio increases as ϕ increases until graphite starts to
 709 form. The CO mole fraction contours for cases 2-1 and 2-8 are shown in Figure 11.

710



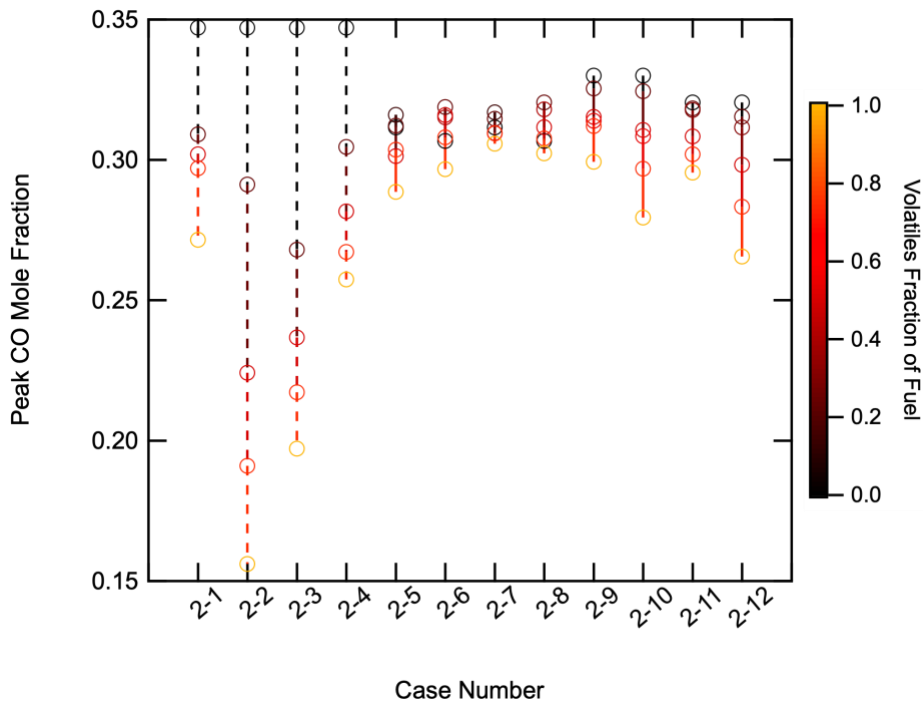
711

712 Figure 11. Equilibrium CO mole fraction calculations by Cantera. Note that all plots have the
 713 same color range, as shown in the color bar in the upper right corner of the plot.

714

715 Because of the lower oxygen content, CO becomes more prevalent in fuel rich
 716 conditions. There is very little difference in the shape of these contours between the CO mole
 717 fraction from tar properties compared to the CO mole fraction of total volatiles properties. Like
 718 the CO₂ mole fractions, the differences become slightly more pronounced with higher volatiles
 719 mixtures, particularly in the gradient, or how fast CO increases with increasing equivalence ratio.

720 There is a significant difference between the calculated CO of the coal-based fuels and
 721 the simplified surrogate gases, as seen for the CO₂ mole fractions. In mixtures that are almost all
 722 graphite, there is a larger range of very high CO mole fractions. However, with mostly volatile
 723 surrogates, the CO mole fraction is much smaller than with coal-based fuels. The peak CO mole
 724 fraction ranges for each fuel case are shown in Figure 12.
 725



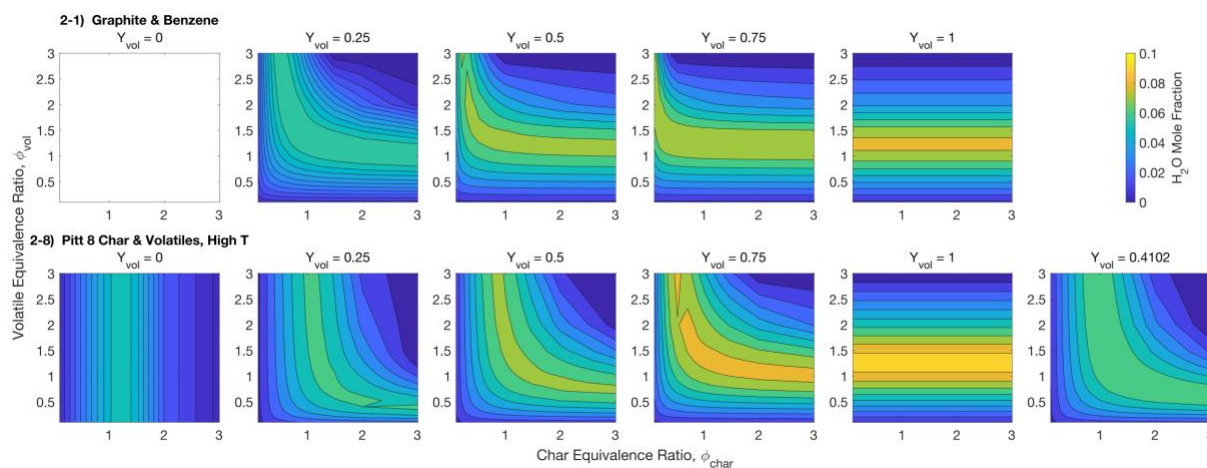
726
 727 Figure 12. Peak CO mole fraction ranges for all fuel cases listed in Table 4. Dashed lines
 728 correspond to simple surrogate gases and the circles correspond to each fuel mixing condition.
 729 The color of the lines and dots depends on the fuel mixing ratio (Y_{vol} ranging from 0 to 100%
 730 volatiles).
 731

732 The peak CO mole fraction ranges follow a very similar trend as the CO₂ mole fractions,
 733 although with even greater ranges, from 15 to 35 mol%. The peak CO mole fractions, however,

734 occur only in extreme fuel-rich conditions. This trade-off between the locations of greatest CO
735 and CO₂ mole fractions is because the ratio of carbon dioxide and carbon monoxide are based on
736 the available oxygen and temperature of the gas environment.

737 In the one-mixture fraction comparison, the H₂O mole fraction was very different
738 depending on what fuel was used in the equilibrium calculations. The H₂O mole fraction
739 contours for the two-mixture fraction comparison are given in Figure 13.

740



741

742 Figure 13. Equilibrium H₂O mole fractions as calculated by Cantera. Note that the plots for pure
743 graphite show as a blank plot (top left) because graphite has no hydrogen to form H₂O.

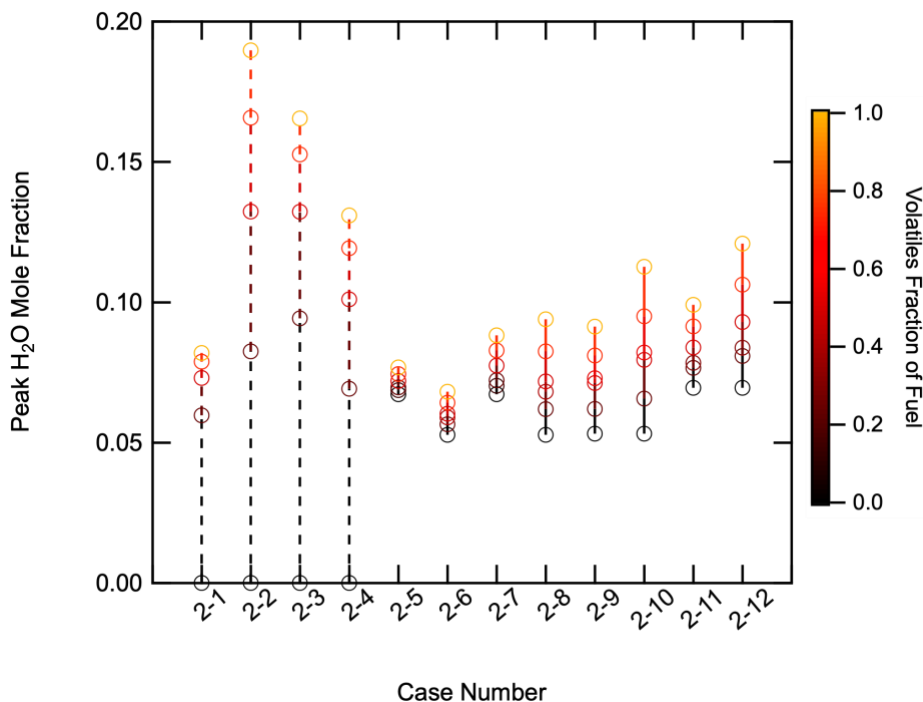
744

745 There is a significant difference in the H₂O mole fraction when using tar properties
746 compared to total volatiles, and an even greater difference when using surrogate gases. It is
747 worth noting in the “char only” (first column) mixtures of the surrogate fuels, the plots appear to
748 be blank. This is not a mistake, rather, the fuel mixture is entirely graphite in those equilibrium
749 states, and therefore there is no fuel hydrogen to contribute to an equilibrium moisture content.
750 There is no additional hydrogen in the air, so the H₂O mole fraction at equilibrium is entirely
751 dependent on the hydrogen content of the original fuel. In addition, the moisture content of the

752 equilibrated mixture highly influences both gasification reactions and radiative heat transfer in
753 the combustion system.

754 The peak H₂O mole fraction ranges are shown in Figure 14. The peak H₂O mole fraction
755 is highly dependent on the fuel chosen (similar to the one-mixture fraction results) and can range
756 from no H₂O produced (graphite) to almost 20 mol% produced (methane). Fuels with a higher
757 hydrogen fraction end up with a higher equilibrium moisture content than fuels with less
758 hydrogen. In coal pyrolysis, the volatiles tend to become enriched in hydrogen, especially in the
759 light gas species that include compounds like methane (CH₄), ethane (C₂H₆), ethylene (C₂H₄)
760 and other simple hydrocarbons. Accurately determining where fuel hydrogen is and when it is
761 accessible to gas-phase chemistry is very important in determining other heterogeneous reactions
762 as well as radiative heat transfer. This is not as important with a one-mixture fraction system but
763 becomes more important in two- or three-mixture fraction systems.

764



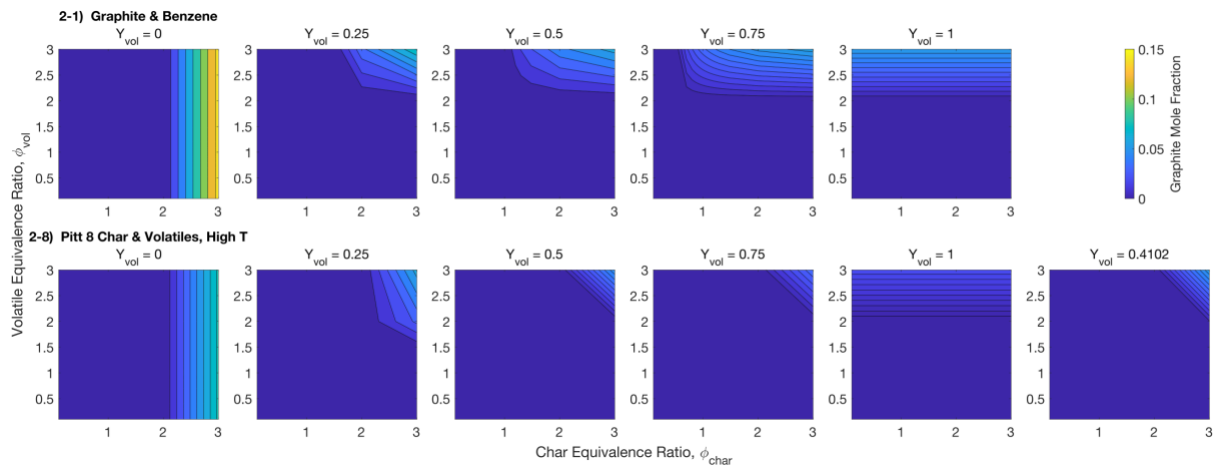
765

766 Figure 14. Peak H₂O mole fraction ranges for all two-mixture fraction fuel cases listed in Table
 767 4. Dashed lines correspond to simple surrogate gases and the circles correspond to each fuel
 768 mixing condition. The color of the lines and dots depends on the fuel mixing ratio (Y_{vol} ranging
 769 from 0 to 100% volatiles).

770

771 Graphite (solid carbon) was shown in the one-mixture fraction approach to only matter in
 772 fuel-rich conditions. Industrial combustion applications usually do not operate in such conditions
 773 because it usually means some of the original fuel is not being completely burned, which can
 774 greatly impact many aspects of a combustion apparatus. The graphite mole fraction contours for
 775 cases 2-1 and 2-8 are shown in Figure 15.

776



777

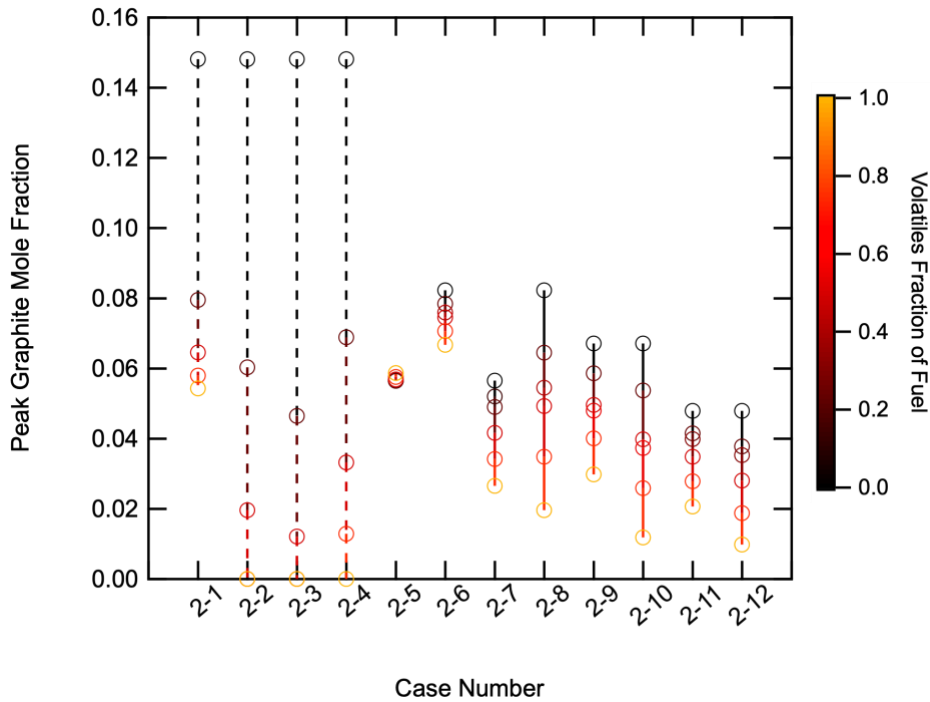
778 Figure 15. Equilibrium graphite mole fractions as calculated by Cantera. Note that all plots have
 779 the same color range, as shown in the color bar in the upper right corner of the plot.

780

781 Like the one-mixture fraction comparison, many of the fuel mixtures in the two-mixture
 782 fraction comparison have very little graphite formation and only in the extreme fuel-rich

783 locations. The peak graphite mole fraction ranges for each two-mixture fraction case are shown
 784 in Figure 16.

785



786

787 Figure 16. Peak graphite mole fraction ranges for all two-mixture fraction fuel cases listed in
 788 Table 4. Dashed lines correspond to simple surrogate gases and the circles correspond to each
 789 fuel mixing condition. The color of the lines and dots depends on the fuel mixing ratio (Y_{vol}
 790 ranging from 0 to 100% volatiles).

791

792 The ranges of peak graphite mole fractions are quite varied, with the largest ranges in the
 793 surrogate gases, ranging between no equilibrium graphite to 15 mol% in the pure graphite fuel
 794 mixtures. None of the coal-based fuels exceed 9 mol% graphite at equilibrium, and many of the
 795 fuel cases fall well below that percentage. Incorrectly calculating solid carbon at equilibrium

796 could cause major errors in large-scale simulations, particularly in deciding how much over-fire
797 air might be needed to completely burn out the original fuel.

798 A two-mixture fraction approach seems to be more appropriate for coal-based fuels than
799 a one-mixture fraction approach. In an industrial coal boiler, the coal particles enter the hot
800 environment (typically in slightly fuel rich conditions) and almost immediately begin to
801 pyrolyze. The whole pyrolysis process typically lasts milliseconds with the high particle heating
802 rates of pulverized coal particles. This means that the local gas composition is a mix of oxidizer
803 and volatile gases from the coal. The char remains a solid until the reactive gases can make it to
804 the char surface and begin to react, adding mass from the char into the gas mixture. This means
805 that coal pyrolysis and char conversion can be easily described using separate mixture fractions.
806 Care must be taken when choosing the compositions and heating values of the volatiles and char.
807 The results here indicate that choosing a simple volatile or char surrogate gas as a direct
808 replacement for measured coal properties results in large differences in the equilibrium mole
809 fractions of CO₂, CO, and H₂O. Much of this difference in equilibrium composition can be traced
810 back to the difference in elemental compositions between fuels, as illustrated in Table 2. This is
811 one of the key reasons why more than one mixture fraction is needed to accurately describe coal
812 combustion reactions.

813

814 3.3 Three Mixture Fractions

815 Because coal pyrolysis results in three main products (char, tar, and light gas), coal
816 pyrolysis can be easily divided into a three-mixture fraction approach. There was only one coal
817 that included enough information to calculate an average light gas composition and heating
818 value—the Pitt 8 coal, specifically with low temperature pyrolysis products (char, tar, and light

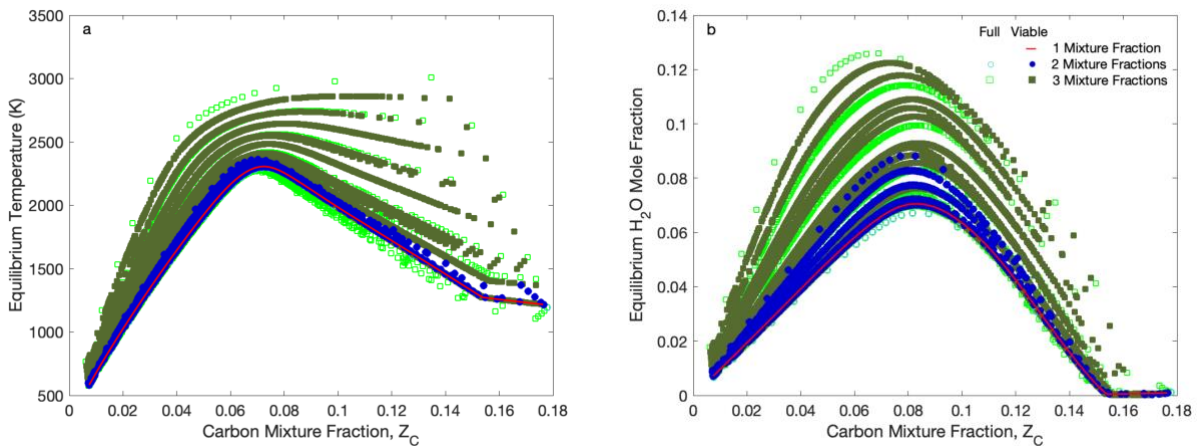
819 gases). Because only one fuel was used to calculate equilibrium states based on a three-mixture
820 fraction system, the results for this fuel were compared against the results for the same fuel using
821 both one and two mixture fractions.

822 The procedure used to set up the three-mixture fraction comparison is described in
823 greater detail in the introduction and approach sections; however, it is similar to the approach
824 used in the two-mixture fraction comparisons, with the fuel mixing conditions being slightly
825 more complex. In this case, the fractions of char and tar were allowed to vary between 0 and 1,
826 including the fractions of each at the reported pyrolysis conditions. The fraction of light gas was
827 the remainder of the fuel mixture, and the sum of the three fuel fractions was constrained to sum
828 to one. The oxidizer was handled the same way here as in the one- and two-mixture fraction
829 approaches. The oxidizer contribution for each fuel stream was allowed to vary independently in
830 an equivalence ratio between 0.1 and 3, with all stream contributions weighted by the fractions
831 of each fuel stream.

832 The one-mixture fraction approach calculated at about 20 different equilibrium states
833 over the whole range of equivalence ratios for each fuel; the two-mixture fraction approach
834 calculated around 2,000 equilibrium states for each fuel; and the three-mixture fraction approach
835 calculated almost 150,000 equilibrium states for a single fuel. The one-mixture fraction approach
836 finished in generally under a minute for each fuel; the two-mixture fraction approach took
837 anywhere from an hour to a couple of days in Cantera; and the three-mixture fraction approach
838 took four or more days on a single processor.

839 Because of the large amount of data generated in the three-mixture fraction comparison,
840 it would be difficult to compare the results of the coal-based fuels to the simple surrogate gases
841 in any meaningful way. A more meaningful approach compares the results of all three mixture

842 fraction approaches for one fuel. The same key variables were used to compare all three mixture
 843 fractions together, starting with the equilibrium temperature and H₂O mole fraction shown in
 844 Figure 17. To compare similar values, all of these comparison plots are made with respect to the
 845 carbon mixture fraction (Z_c) rather than any of the component mixture fractions (f_i) or
 846 equivalence ratios. For comparisons using the same parent fuel, any individual elemental mixture
 847 fraction, Z_j , would have the same range regardless of the number of component mixture fractions
 848 used, while allowing for a comparison of all mixture fraction approaches on one figure in one
 849 mixture fraction dimension. In this comparison, the carbon mixture fraction seemed to be the
 850 most appropriate to use since the bulk of the fuel is comprised of carbon.
 851



852
 853 Figure 17. (a) Equilibrium temperature and (b) H₂O mole fraction calculations by Cantera for the
 854 Pitt 8 coal using one (case 1-6), two (case 2-7), and three mixture fractions. Note that the two-
 855 and three-mixture fraction results are shown as “viable” and “full.” The viable results only
 856 including results for mixtures of fuels with the maximum char yield based on pyrolysis
 857 measurements.
 858

859 Two sets of results are displayed for both the two- and three-mixture fraction approaches.
860 The “full” results include all equilibrium states over the full range of fuel mixing conditions and
861 the “viable” results limit the fuel mixing conditions to have a maximum char yield based on
862 pyrolysis measurements. In combustion applications, the other pyrolysis products (tar and light
863 gas) enter the gas phase before the solid char begins to react with the gas phase. For this reason,
864 there are situations where there might be equilibrium between the volatile gases (tar and light
865 gases) before any material from the char mixes in with the gas phase, but the reverse situation
866 (i.e., all char and no pyrolysis gases) would not occur unless the starting fuel was only char.

867 The peak temperature changes with the fuel mixing condition, especially in the three-
868 mixture fraction approach. Each distinct “line” of data points in the three-mixture fraction results
869 corresponds to a different fuel mixing condition (i.e., a different set of f_i 's). The temperatures for
870 the one- and two-mixture fraction approaches (Figure 17a) are very close to each other, with the
871 two-mixture fraction temperatures having a slightly increased range (shown here as a wider
872 temperature band). The three-mixture fraction approach, however, allows for much hotter
873 temperatures (up to almost 3,000 K) in addition to some fuel mixtures with similar temperatures
874 to both the one- and two-mixture fraction approaches. The hotter temperatures achieved by the
875 three-mixture fraction approach correspond to mixtures with greater percentages of the light gas
876 species, which would include some of the same simple hydrocarbon surrogates with much higher
877 temperatures shown in the one-mixture fraction comparison. Both the two- and three-mixture
878 fraction results include many more equilibrium states than the one-mixture fraction results.

879 In the one- and two-mixture fraction approaches, the H₂O mole fraction was highly
880 dependent on the fuel used. This result was also seen in the three-mixture fraction approach (see
881 Richards [70]), although the only case shown in Figure 17 is for a coal with realistic

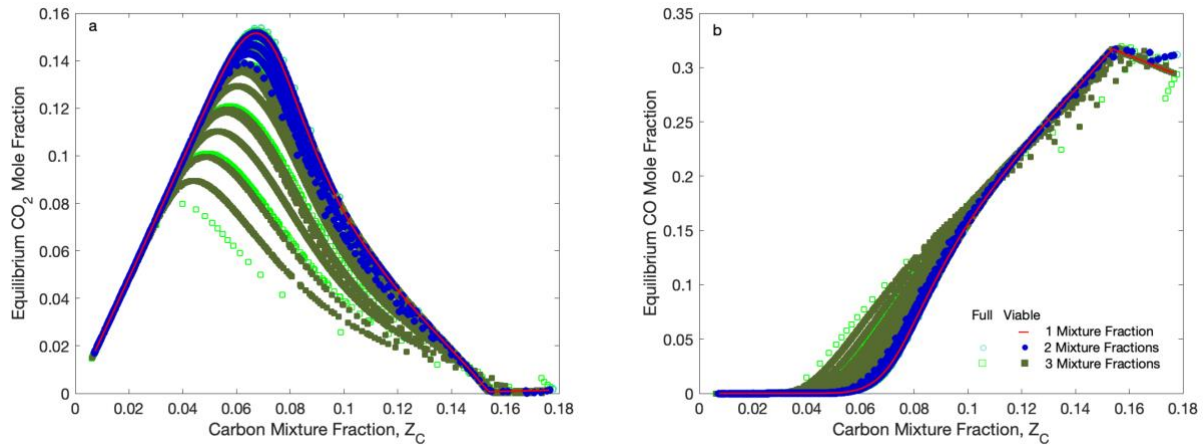
882 compositions and heating values of char, tar, and light gas. Note that this set of data was from a
883 low temperature pyrolysis experiment. The H₂O mole fraction is highly dependent on the fuel
884 mixing conditions (how much of char, tar, and light gases are mixed into the overall reactant
885 mix). The peak H₂O mole fraction in Figure 17 varies between 6 and 13 mol% for the three-
886 mixture fraction approach and between 6 and 8 mol% using two mixture fractions. This large
887 variability might be enough to cause large errors in large-scale simulations if fewer mixture
888 fractions or even simple coal surrogate gases are used in the place of more complex mixture
889 fraction approaches.

890 Moisture can play a large role in large-scale combustion simulations, contributing to
891 additional chemical reactions (mainly gasification) and impacting radiative heat transfer
892 calculations. This was discussed to some extent in the two-mixture fraction comparison,
893 however, in a three-mixture fraction system, knowing where hydrogen is in a system is even
894 more important. The light gas components tend to be much more enriched in hydrogen than
895 either the char or the tar. This is one of the reasons for the large variability in the equilibrium
896 H₂O mole fraction in a three-mixture fraction system as opposed to a one- or two-mixture
897 fraction system.

898 The equilibrium O₂ mole fraction was very similar over all fuel mixing conditions even in
899 the three-mixture fraction calculations, and are not shown here (see Richards [70]). The CO₂
900 mole fraction had a wide range in both the one- and two-mixture fraction comparisons,
901 depending on the fuel choice. Figure 18 shows the comparison of the one-, two-, and three-
902 mixture fraction calculations of the mole fractions of CO₂ and CO for the Pitt 8 coal. The carbon
903 in the fuel is the only source of carbon in these equilibrium calculations, so small changes in the
904 total amount of each fuel component greatly influence the overall amount of CO₂ formed at

905 equilibrium. Including light gases as a separate fuel stream changes the amount of CO₂ even
906 more since the carbon content is greatly reduced in light gas components that tend to be enriched
907 in hydrogen (e.g., methane) or oxygen (like CO or CO₂). The three-mixture fraction calculations
908 show that CO becomes more prominent in fuel-rich conditions where the carbon cannot fully
909 oxidize to CO₂. This result is similar to the findings from the one- and two-mixture fraction
910 calculations. However, the CO mole fraction also tends to be a little more spread out in the three-
911 mixture fraction approach for Z_c values ranging from 0.04 to 0.09.

912



913

914 Figure 18. (a) Equilibrium CO₂ and (b) CO mole fraction calculations by Cantera for the Pitt 8
915 coal using one, two, and three mixture fractions.

916

917 In summary, the three-mixture fraction approach results in much more variety in
918 equilibrium states than a simpler one- or two-mixture fraction approach, which would likely
919 increase overall accuracy of coal combustor simulations using gas-phase equilibrium
920 calculations. However, this greater variety would of necessity include significantly greater
921 computational time and complexity. A two-mixture fraction approach that combines the tar and

922 light gases into a single “volatiles” mixture fraction would give greater variability and accuracy
923 than a single coal mixture fraction while not greatly increasing computational complexity.

924

925 3.4 Implications for Large Simulations

926 Carbon dioxide and water are two gases that greatly impact radiative heat transfer. Both
927 of these gases are highly influenced by both the original fuel used and the number of mixture
928 fractions used. The equilibrium H₂O mole fraction in particular varies widely from zero with
929 pure graphite in the one- and two-mixture fraction approaches to almost 20 mol% when using
930 only methane as a surrogate gas in place of coal. Even when using measured values from coal
931 and coal-based fuels there is a moderate amount of variability in both CO₂ and H₂O mole
932 fractions, regardless of the number of mixture fractions used.

933 While a three-mixture fraction approach increases the variability of equilibrium states, it
934 also greatly increases the computational time and complexity, often more than is desirable for
935 large-scale simulations. In large-scale simulations, equilibrium calculations are often used to
936 generate lookup tables to use in calculating the gas chemistry reactions, which means most of the
937 time spent on equilibrium calculations would be incurred at the beginning of a simulation. When
938 parallelized, the equilibrium calculations likely would not be significant compared to the overall
939 simulation time, potentially only being an issue in smaller simulations. A two-mixture fraction
940 approach that splits coal fuels into char and total volatiles can strike a better compromise
941 between increased variability and accuracy while keeping the computational complexity low. In
942 industrial applications, pyrolysis often occurs so fast that it is hard to distinguish a difference in
943 when tars and light gases enter the gas phase.

944 One of the conclusions from Flores and Fletcher [50] was that their two-mixture fraction
945 approach would be better than a one-mixture fraction analysis in accurately accounting for soot
946 precursors. Tar was modeled as a constant fraction of the volatiles. In coal combustion systems,
947 the primary source of soot precursors is the tar [71, 72]. A two-mixture fraction approach would
948 be much better than a one-mixture fraction approach for modeling tar precursors, but a three-
949 mixture fraction approach that models tar separately from light gases would seem to be most
950 beneficial in calculating soot precursors. Hybrid approaches have been used where a mixture
951 fraction of tar is solved for soot formation purposes but not fully integrated into the gas-phase
952 chemistry calculations [71, 72].

953 The use of two or three fuel mixture fractions would not affect the outlet equilibrium
954 composition of a combustor or gasifier if the coal achieved 100% conversion. The region of the
955 combustor that would be most affected would be near the burner after pyrolysis but before
956 significant char combustion or gasification occurred. This near-burner region plays a significant
957 role in the formation of pollutants such as NO_x and soot, and therefore accurate calculations in
958 this region are very important.

959 All mixture fraction methods used here were based on complete equilibrium. While
960 equilibrium can be a useful assumption in combustion systems, it might not be the best
961 assumption to use in coal-based systems. Other such assumptions might include only products of
962 complete combustion, or by substituting the water-gas shift reaction in place of complete
963 equilibrium. It would be appropriate to test these different reaction assumptions using the
964 mixture fraction methods discussed here in order to quantify any uncertainty in reaction
965 assumptions, especially when applied to coal-based fuels.

966

967 4. Conclusions and Recommendations

968 The one-mixture fraction comparison showed that the NASA-CEA and Cantera
969 equilibrium programs generally agree when using both simple hydrocarbon surrogate gases and
970 more complex coal-based fuels. However, there was some significant deviation between the two
971 programs with some of the high-rank coals (some anthracites and semi-anthracites) in very fuel-
972 rich conditions ($\phi > 2$). These deviations occur due to the way Cantera was set up to handle
973 unreacted fuel in order to minimize numerical instabilities in the equilibrium calculations. Both
974 NASA and most of the Cantera equilibrium states convert the unreacted fuel to graphite, but in
975 the few cases where deviation occurred, Cantera instead left the unreacted fuel with the same
976 composition and properties of the original fuel. These deviations might be made smaller with
977 improvements to the solid coal Cantera mechanism subroutine to better account for higher rank
978 coals. While most industrial combustion applications do not use either high rank coals or such
979 fuel-rich conditions on an overall basis, the local stoichiometry likely will fall to the fuel-rich
980 extreme, especially in pyrolysis or gasification conditions. This means that more accurate
981 modeling of fuel-rich conditions will be necessary for highly detailed, large-scale simulations of
982 coal combustion systems.

983 In addition, the one-mixture fraction comparison showed that the equilibrium temperature
984 and mole fractions of CO_2 and H_2O are highly dependent on the fuel used, and the CO mole
985 fraction is highly dependent on the fuel only in fuel-rich conditions. In a one-mixture fraction
986 system, this difference is nothing more than the difference in the elemental compositions of the
987 original fuels, and this might be largely mitigated by using a combination of different surrogate
988 gases and reference temperatures to match both composition and enthalpy of measured coal-
989 based fuels. This procedure might cause the gas temperature to be out of the range of normal

990 combustion conditions or even to be an impossible temperature. However, using the surrogate
991 gases individually as direct replacements for coal-based fuels will make large-scale simulations
992 inaccurate for any combination of mixture fractions. This is especially true when trying to
993 accurately model gas compositions for radiative heat transfer calculations, which are greatly
994 influenced by both CO₂ and H₂O compositions. Not only do the gas compositions greatly affect
995 the heat transfer processes, but also impact many heterogeneous and homogenous reaction rates,
996 both in terms of actual reaction speed and diffusion speed. As a final note on the one-mixture
997 fraction analysis, the equilibrium oxygen content was not significantly influenced by fuel choice,
998 but more by the ratio of air to fuel. The oxygen content did not significantly change even for
999 coal-based fuels that can include a moderate amount of oxygen bound in the organic matrix.

1000 The two-mixture fraction comparison showed similar results to the one-mixture fraction
1001 approach for equilibrium temperature and mole fractions of CO₂, H₂O, and CO, especially in the
1002 limiting cases of using only one fuel. Both the CO and H₂O peak mole fractions varied by up to
1003 20 mol% based on fuel type, while the peak CO₂ mole fraction varied by closer to 10 mol%. The
1004 peak temperature varied between 2,200 and 2,400 K, depending on the fuel. In addition, the two-
1005 mixture fraction approach offers a broader range of values for the equilibrium temperature and
1006 mole fractions for the different combinations of component mixture fractions than the one-
1007 mixture fraction approach. While the two-mixture fraction approach does take more time, this
1008 approach is closer to what occurs in a real coal combustion system during devolatilization. The
1009 total volatiles enter the gas phase earlier than gases from the char reactions, and the tar and the
1010 light gases seem to be released at similar times in the pyrolysis process which is consistent with a
1011 two-mixture fraction approach. The O₂ mole fraction at equilibrium did not change much with
1012 respect to fuel choice in a manner similar to the one-mixture fraction approach. The peak O₂

1013 mole fraction only varied by just over 0.2 mol% across all fuels in the two-mixture fraction
1014 comparison, with most of that variability in the coal surrogate fuels.

1015 Both the one- and two-mixture fraction calculations showed that using measured coal-
1016 based fuel properties will always be more accurate than using coal surrogate gases (like graphite,
1017 benzene, methane, or other simple hydrocarbons or their combinations) as a direct substitution
1018 for coal-based fuels, especially in accurately modeling the CO₂, CO and H₂O mole fractions.

1019 While the three-mixture fraction method offers a broader range of values in equilibrium
1020 temperatures and gas compositions than either the one- or two-mixture fraction methods, it is
1021 unclear whether a three-mixture fraction system is more realistic than a well-designed two-
1022 mixture fraction method. The two-mixture fraction method, if well designed, will allow for
1023 enough variability in fuel properties to come close to real coal reactions without adding too much
1024 complexity. Since the tar and light gases are released at similar times during coal pyrolysis, it is
1025 easy to justify a two-mixture fraction method, especially in simulations that do not have small
1026 enough time steps to differentiate between tar and light gas release. There is potential benefit,
1027 however, in using a three-mixture fraction approach in modeling coal combustion when trying to
1028 accurately model soot and soot precursors, since most of the soot in coal systems comes from the
1029 tar.

1030

1031 Acknowledgements

1032 This work was supported in part by the Department of Energy, National Nuclear Security
1033 Administration, under Award Number DE-NA0002375.

1034

1035 References

- 1036 [1] Smoot, L.D., Smith, P.J., Coal Combustion and Gasification, Plenum Press, New York, 1985.
- 1037 [2] Chen, L., Yong, S.Z., Ghoniem, A.F., Oxy-Fuel Combustion of Pulverized Coal:
1038 Characterization, Fundamentals, Stabilization and CFD Modeling, Progress in Energy and
1039 Combustion Science, 2012;38:156-214.
- 1040 [3] Pedel, J., Thornock, J.N., Smith, P.J., Ignition of Co-axial Turbulent Diffusion Oxy-coal Jet
1041 Flames: Experiments and Simulations Collaboration, Combustion and Flame,
1042 2013;160:1112-1128.
- 1043 [4] Smith, K.L., Smoot, L.D., Fletcher, T.H., Pugmire, R.J., The Structure and Reaction
1044 Processes of Coal, Plenum Press, New York, 1994.
- 1045 [5] Barnhart, J.S., Laurendeau, N.M., Pulverized Coal Combustion and Gasification in a Cyclone
1046 Reactor .1. Experiment, Industrial & Engineering Chemistry Process Design and
1047 Development, 1982;21:671-680.
- 1048 [6] Barnhart, J.S., Thomas, J.F., Laurendeau, N.M., Pulverized Coal Combustion and
1049 Gasification in a Cyclone Reactor .2. Model and Comparison With Experiment, Industrial
1050 & Engineering Chemistry Process Design and Development, 1982;21:681-689.
- 1051 [7] Bhuiyan, A.A., Naser, J., Numerical Modelling of Oxy Fuel Combustion, the Effect of
1052 Radiative and Convective Heat Transfer and Burnout, Fuel, 2015;139:268-284.
- 1053 [8] Bhunia, S., Sadhukhan, A.K., Haldar, S., Mondal, P.P., Prabhakar, A., Gupta, P.,
1054 Devolatilization and Combustion of Coarse-Sized Coal Particles in Oxy–Fuel Conditions:
1055 Experimental and Modeling Studies, Energy & Fuels, 2018;32:839-854.
- 1056 [9] Borghi, G., Sarofim, A.F., Beér, J.M., A Model of Coal Devolatilization and Combustion in
1057 Fluidized Beds, Combustion and Flame, 1985;61:1-16.

- 1058 [10] Vascellari, M., Roberts, D.G., Hla, S.S., Harris, D.J., Hasse, C., From Laboratory-Scale
1059 Experiments to Industrial-Scale CFD Simulations of Entrained Flow Coal Gasification,
1060 Fuel, 2015;152:58-73.
- 1061 [11] Rieth, M., Proch, F., Clements, A.G., Rabaçal, M., Kempf, A.M., Highly Resolved Flamelet
1062 LES of a Semi-Industrial Scale Coal Furnace, Proc. Combust. Inst., 2017;36:3371-3379.
- 1063 [12] Wen, X., Luo, K., Wang, H.O., Luo, Y.J., Fan, J.R., Analysis of Pulverized Coal Flame
1064 Stabilized in a 3D Laminar Counterflow, Combustion and Flame, 2018;189:106-125.
- 1065 [13] Backreedy, R.I., Habib, R., Jones, J.M., Pourkashanian, M., Williams, A., An Extended
1066 Coal Combustion Model, Fuel, 1999;78:1745-1754.
- 1067 [14] Bradley, D., Lawes, M., Park, H.Y., Usta, N., Modeling of Laminar Pulverized Coal Flames
1068 With Speciated Devolatilization and Comparisons With Experiments, Combustion and
1069 Flame, 2006;144:190-204.
- 1070 [15] Tufano, G.L., Stein, O.T., Kronenburg, A., Gentile, G., Stagni, A., Frassoldati, A., Faravelli,
1071 T., Kempf, A.M., Vascellari, M., Hasse, C., Fully-Resolved Simulations of Coal Particle
1072 Combustion Using a Detailed Multi-Step Approach for Heterogeneous Kinetics, Fuel,
1073 2019;240:75-83.
- 1074 [16] Van Essendelft, D., Li, T.W., Nicoletti, P., Jordan, T., Advanced Chemistry Surrogate
1075 Model Development within C3M for CFD Modeling, Part 1: Methodology Development
1076 for Coal Pyrolysis, Ind. Eng. Chem. Res., 2014;53:7780-7796.
- 1077 [17] Fletcher, T.H., Hardesty, D.R., Compilation of Sandia Coal Devolatilization Data:
1078 Milestone Report, Sandia National Laboratories, Livermore, CA, 1992.

- 1079 [18] Pielsticker, S., Gövert, B., Kreitzberg, T., Habermehl, M., Hatzfeld, O., Kneer, R.,
1080 Simultaneous Investigation Into the Yields of 22 Pyrolysis Gases From Coal and Biomass
1081 in a Small-Scale Fluidized Bed Reactor, *Fuel*, 2017;190:420-434.
- 1082 [19] Al-Abbas, A.H., Naser, J., Hussein, E.K., Numerical Simulation of Brown Coal Combustion
1083 in a 550 MW Tangentially-Fired Furnace Under Different Operating Conditions, *Fuel*,
1084 2013;107:688-698.
- 1085 [20] Costa, V.J., Krioukov, V.G., Maliska, C.R., Numerical Simulation of Pulverized Wet Coal
1086 Combustion Using Detailed Chemical Kinetics, *J. Braz. Soc. Mech. Sci. Eng.*,
1087 2014;36:661-672.
- 1088 [21] Cui, K., Liu, B., Wu, Y.X., Yang, H.R., Lu, J.F., Zhang, H., Numerical Simulation of Oxy-
1089 coal Combustion for a Swirl Burner With EDC Model, *Chin. J. Chem. Eng.*, 2014;22:193-
1090 201.
- 1091 [22] Guo, J.J., Liu, Z.H., Wang, P., Huang, X.H., Li, J., Xu, P., Zheng, C.G., Numerical
1092 Investigation on Oxy-Combustion Characteristics of a 200 MW_e Tangentially Fired Boiler,
1093 *Fuel*, 2015;140:660-668.
- 1094 [23] Hashimoto, N., Shirai, H., Numerical Simulation of Sub-Bituminous Coal and Bituminous
1095 Coal Mixed Combustion Employing Tabulated-Devolatilization-Process Model, *Energy*,
1096 2014;71:399-413.
- 1097 [24] Jovanovic, R., Milewska, A., Swiatkowski, B., Goanta, A., Spliethoff, H., Sensitivity
1098 Analysis of Different Devolatilisation Models on Predicting Ignition Point Position During
1099 Pulverized Coal Combustion in O₂/N₂ and O₂/CO₂ Atmospheres, *Fuel*, 2012;101:23-37.

- 1100 [25] Lisandy, K.Y., Kim, R.G., Hwang, C.W., Jeon, C.H., Sensitivity Test of Low Rank
1101 Indonesian Coal Utilization Using Steady State and Dynamic Simulations of Entrained-
1102 Type Gasifier, *Appl. Therm. Eng.*, 2016;102:1433-1450.
- 1103 [26] Wen, X., Luo, K., Luo, Y.J., Kassem, H.I., Jin, H.H., Fan, J.R., Large Eddy Simulation of a
1104 Semi-Industrial Scale Coal Furnace Using Non-Adiabatic Three-Stream Flamelet/Progress
1105 Variable Model, *Appl. Energy*, 2016;183:1086-1097.
- 1106 [27] Watanabe, J., Okazaki, T., Yamamoto, K., Kuramashi, K., Baba, A., Large-Eddy Simulation
1107 of Pulverized Coal Combustion Using Flamelet Model, *Proc. Combust. Inst.*,
1108 2017;36:2155-2163.
- 1109 [28] Watanabe, J., Yamamoto, K., Flamelet Model for Pulverized Coal Combustion, *Proc.*
1110 *Combust. Inst.*, 2015;35:2315-2322.
- 1111 [29] Rabaçal, M., Franchetti, B.M., Marincola, F.C., Proch, F., Costa, M., Hasse, C., Kempf,
1112 A.M., Large Eddy Simulation of Coal Combustion in a Large-Scale Laboratory Furnace,
1113 *Proc. Combust. Inst.*, 2015;35:3609-3617.
- 1114 [30] Vascellari, M., Tufano, G.L., Stein, O.T., Kronenburg, A., Kempf, A.M., Scholtissek, A.,
1115 Hasse, C., A Flamelet/Progress Variable Approach for Modeling Coal Particle Ignition,
1116 *Fuel*, 2017;201:29-38.
- 1117 [31] Veras, C.A.G., Saastamoinen, J., Carvalho, J.A., Aho, N., Overlapping of the
1118 Devolatilization and Char Combustion Stages in the Burning of Coal Particles, *Combustion*
1119 *and Flame*, 1999;116:567-579.
- 1120 [32] Messig, D., Vascellari, M., Hasse, C., Flame Structure Analysis and Flamelet Progress
1121 Variable Modelling of Strained Coal Flames, *Combust. Theory Model.*, 2017;21:700-721.

1122 [33] Backreedy, R.I., Fletcher, L.M., Ma, L., Pourkashanian, M., Williams, A., Modelling
1123 Pulverised Coal Combustion Using a Detailed Coal Combustion Model, *Combust. Sci.*
1124 *Technol.*, 2006;178:763-787.

1125 [34] Franchetti, B.M., Marincola, F.C., Navarro-Martinez, S., Kempf, A.M., Large Eddy
1126 Simulation of a Pulverised Coal Jet Flame, *Proc. Combust. Inst.*, 2013;34:2419-2426.

1127 [35] Rebola, A., Azevedo, J.L.T., Modelling Pulverized Coal Combustion Using Air and O₂⁺
1128 Recirculated Flue Gas as Oxidant, *Appl. Therm. Eng.*, 2015;83:1-7.

1129 [36] Chen, L., Ghoniem, A.F., Simulation of Oxy-Coal Combustion in a 100 kW_{th} Test Facility
1130 Using RANS and LES: A Validation Study, *Energy & Fuels*, 2012;26:4783-4798.

1131 [37] Theron, J.A., le Roux, E., Representation of Coal and Coal Derivatives in Process
1132 Modelling, *Journal of the Southern African Institute of Mining and Metallurgy*,
1133 2015;115:339-348.

1134 [38] Solomon, P.R., Fletcher, T.H., Pugmire, R.J., *Progress in Coal Pyrolysis*, *Fuel*,
1135 1993;72:587-597.

1136 [39] Fletcher, T.H., Review of 30 Years of Research Using the Chemical Percolation
1137 Devolatilization Model, *Energy & Fuels*, 2019;33:12123-12153.

1138 [40] Richards, A.P., Johnson, C., Fletcher, T.H., Correlations of the Elemental Compositions of
1139 Primary Coal Tar and Char, *Energy & Fuels*, 2019;33:9520-9537.

1140 [41] Goshayeshi, B., Sutherland, J.C., A Comparison of Various Models in Predicting Ignition
1141 Delay in Single-Particle Coal Combustion, *Combustion and Flame*, 2014;161:1900-1910.

1142 [42] Xu, H.B., Hunger, F., Vascellari, M., Hasse, C., A Consistent Flamelet Formulation for a
1143 Reacting Char Particle Considering Curvature Effects, *Combustion and Flame*,
1144 2013;160:2540-2558.

- 1145 [43] Knappstein, R., Kuenne, G., Meier, T., Sadiki, A., Janicka, J., Evaluation of Coal Particle
1146 Volatiles Reaction by Using Detailed Kinetics and FGM Tabulated Chemistry, Fuel,
1147 2017;201:39-52.
- 1148 [44] Rieth, M., Kempf, A.M., Stein, O.T., Kronenburg, A., Hasse, C., Vascellari, M., Evaluation
1149 of a Flamelet/Progress Variable Approach for Pulverized Coal Combustion in a Turbulent
1150 Mixing Layer, Proc. Combust. Inst., 2019;37:2927-2934.
- 1151 [45] Brewster, B.S., Baxter, L.L., Smoot, L.D., Treatment of Coal Devolatilization in
1152 Comprehensive Combustion Modeling, Energy & Fuels, 1988;2:362-370.
- 1153 [46] Rieth, M., Proch, F., Rabaçal, M., Franchetti, B.M., Marincola, F.C., Kempf, A.M.,
1154 Flamelet LES of a Semi-Industrial Pulverized Coal Furnace, Combustion and Flame,
1155 2016;173:39-56.
- 1156 [47] Wen, X., Luo, K., Jin, H.H., Fan, J.R., Numerical Investigation of Coal Flamelet
1157 Characteristics in a Laminar Counterflow With Detailed Chemistry, Fuel, 2017;195:232-
1158 242.
- 1159 [48] Stollinger, M., Naud, B., Roekaerts, D., Beishuizen, N., Heinz, S., PDF Modeling and
1160 Simulations of Pulverized Coal Combustion - Part 1: Theory and Modeling, Combustion
1161 and Flame, 2013;160:384-395.
- 1162 [49] Stollinger, M., Naud, B., Roekaerts, D., Beishuizen, N., Heinz, S., PDF Modeling and
1163 Simulations of Pulverized Coal Combustion - Part 2: Application, Combustion and Flame,
1164 2013;160:396-410.
- 1165 [50] Flores, D.V., Fletcher, T.H., The Use of Two Mixture Fractions to Treat Coal Combustion
1166 Products in Turbulent Pulverized-Coal Flames, Combust. Sci. Technol., 2000;150:1-26.

- 1167 [51] Smoot, L.D., Pulverized Coal Diffusion Flames: A Perspective Through Modeling,
1168 Symposium (International) on Combustion, 1981;18:1185-1202.
- 1169 [52] Radulovic, P.T., Ghani, M.U., Smoot, L.D., An Improved Model for Fixed Bed Coal
1170 Combustion and Gasification, Fuel, 1995;74:582-594.
- 1171 [53] Tufano, G.L., Stein, O.T., Kronenburg, A., Frassoldati, A., Faravelli, T., Deng, L., Kempf,
1172 A.M., Vascellari, M., Hasse, C., Resolved Flow Simulation of Pulverized Coal Particle
1173 Devolatilization and Ignition in Air- and O₂/CO₂-Atmospheres, Fuel, 2016;186:285-292.
- 1174 [54] Wen, X., Luo, Y.J., Wang, H.O., Luo, K., Jin, H.H., Fan, J.R., A Three Mixture Fraction
1175 Flamelet Model for Multi-Stream Laminar Pulverized Coal Combustion, Proc. Combust.
1176 Inst., 2019;37:2901-2910.
- 1177 [55] Smith, P.J., Thomas H, F., Smoot, L.D., Model for Pulverized Coal-Fired Reactors,
1178 Symposium (International) on Combustion, 1981;18:1285-1293.
- 1179 [56] Turns, S.R., An Introduction to Combustion: Concepts and Applications, 3rd ed., McGraw
1180 Hill, New York, 2012.
- 1181 [57] Bilger, R.W., Conditional Moment Closure for Turbulent Reacting Flow, Physics of Fluids
1182 a-Fluid Dynamics, 1993;5:436-444.
- 1183 [58] Bilger, R.W., Stårner, S.H., Kee, R.J., On Reduced Mechanisms for Methane-Air
1184 Combustion in Nonpremixed Flames, Combustion and Flame, 1990;80:135-149.
- 1185 [59] Peters, N., Turbulent Combustion, Cambridge University Press, Cambridge, 2000, p.
- 1186 [60] Fox, R.O., Computational Models for Turbulent Reacting Flows, Cambridge University
1187 Press, Cambridge, 2003.
- 1188 [61] Poinot, T., Veynante, D., Theoretical and Numerical Combustion, 2nd ed., Edwards,
1189 Philadelphia, 2005.

- 1190 [62] Gordon, S., McBride, B.J., Computer Program for Calculation of Complex Chemical
1191 Equilibrium Compositions and Applications I. Analysis., NASA RP 1311, NASA, USA,
1192 1994.
- 1193 [63] McBride, B.J., Gordon, S., Computer Program for Calculation of Complex Chemical
1194 Equilibrium Compositions and Applications II. Users Manual and Program Description.,
1195 NASA RP 1311, NASA, USA, 1996.
- 1196 [64] Goodwin, D.G., Speth, R.L., Moffat, H.K., Weber, B.W., Cantera: An object-oriented
1197 software toolkit for chemical kinetics, thermodynamics, and transport processes, Version
1198 2.4.0, 2018, <https://www.cantera.org>.
- 1199 [65] Richards, A.P., Haycock, D., Frandsen, J., Fletcher, T.H., A Review of Coal Heating Value
1200 Correlations With Application to Coal Char, Tar, and Other Fuels, Fuel, 2021;283:118942.
- 1201 [66] Green, D.W., Perry's Chemical Engineers' Handbook, in, McGraw Hill, New York, 1984.
- 1202 [67] Proscia, W.M., Freihaut, J.D., Rastogi, S., Klinzing, G.E., Thermodynamic Properties of
1203 Pulverized Coal During Rapid Heating Devolatilization Processes, DOE/PC/92176—T8, R.
1204 Center, United States Office of the Assistant Secretary for Fossil Energy, USDOE, East
1205 Hartford, Connecticut (United States), U.T. Corporation, 1994.
- 1206 [68] Edwards, J.H., Smith, I.W., Tyler, R.J., CSIRO Flash Pyrolysis Project: Compendium of
1207 Data, Commonwealth Scientific and Industrial Research Organization, North Ryde
1208 (Australia). Div. of Fossil Fuel, 1983.
- 1209 [69] Miller, B.G., 1 - The Chemical and Physical Characteristics of Coal, in: Clean Coal
1210 Engineering Technology (Second Edition), Butterworth-Heinemann, 2017, pp. 3-60.

1211 [70] Richards, A.P., Coal Pyrolysis Models for Use in Massively Parallel Oxyfuel-Fired Boiler
1212 Simulations, Dissertation, Chemical Engineering, Brigham Young University (in
1213 preparation, 2020).

1214 [71] Brown, A.L., Modeling Soot in Pulverized Coal Flames, Masters, Mechanical Engineering,
1215 Brigham Young University (1997).

1216 [72] Josephson, A.J., Gaffin, N.D., Smith, S.T., Fletcher, T.H., Lignell, D.O., Modeling Soot
1217 Oxidation and Gasification with Bayesian Statistics, Energy & Fuels, 2017;31:11291-
1218 11303.

1219