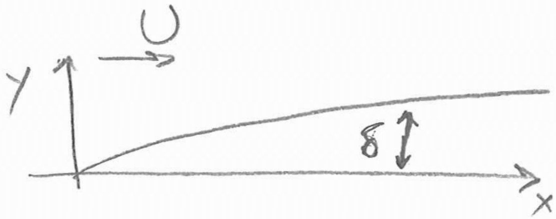
Boundary - Layer Eqns - Flat plate.



$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

BC $\quad y=0 \rightarrow u=v=0$

$\quad\quad y=\infty \rightarrow u=U$

Similarity

- No preferred length
- If scale u by U, y by δ, then the profile looks the same at all x locations.

Then $$\frac{u}{U} = f\left(\frac{y}{\delta}\right) = f(\eta)$$

$$\eta = \frac{y}{\delta} = y\sqrt{\frac{U}{x\nu}}$$

$$\frac{\delta}{L} = \sqrt{\frac{1}{Re}} \rightarrow \delta = \sqrt{\frac{L\nu}{U}} \quad ; \quad L \text{ is } x$$

* (1) $$u = Uf\left(y\sqrt{\frac{U}{x\nu}}\right) = Uf(\eta)$$

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy$$

$$du = U\left[\frac{\partial f}{\partial \eta}\frac{\partial \eta}{\partial x}dx + \frac{\partial f}{\partial \eta}\frac{\partial \eta}{\partial y}dy\right]$$

$$du = U\frac{\partial f}{\partial \eta}\left[-\frac{1}{2}\frac{y}{x}\sqrt{\frac{U}{x\nu}}dx + \sqrt{\frac{U}{x\nu}}dy\right]$$

$$\left.\begin{aligned} &\text{* (2)} \quad \frac{\partial u}{\partial x} = -U\frac{\partial f}{\partial \eta}\cdot\frac{1}{2}\frac{y}{x}\sqrt{\frac{U}{x\nu}} \\ &\text{* (3)} \quad \frac{\partial u}{\partial y} = U\frac{\partial f}{\partial \eta}\sqrt{\frac{U}{x\nu}} \\ &\text{* (4)} \quad \frac{\partial^2 u}{\partial y^2} = \frac{U^2}{x\nu}\frac{d^2 f}{d\eta^2} \end{aligned}\right\}$$

Sub these into the Mom Eqn.

Use Continuity to get v

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x}$$

$$\frac{\partial v}{\partial y} = \frac{Uy}{2x}\sqrt{\frac{U}{x\nu}}\,\frac{df}{d\eta}$$

$$\partial v = \frac{Uy}{2x}\sqrt{\frac{U}{x\nu}}\,\frac{df}{d\eta}\,\partial y \quad ; \quad y = \eta\sqrt{\frac{x\nu}{U}}, \quad \partial y = \sqrt{\frac{x\nu}{U}}\,\partial\eta$$

$$\partial v = \frac{U}{2x}\,\frac{df}{\cancel{d\eta}}\,\cancel{\partial\eta}\cdot\eta\sqrt{\frac{x\nu}{U}}$$

$$\partial v = \frac{U\eta}{2x}\sqrt{\frac{x\nu}{U}}\,df$$

$$\int \longrightarrow \quad v = \frac{U}{2}\sqrt{\frac{\nu}{xU}}\int \eta\,df$$

$$\int u\,dv = uv - \int v\,du$$

$$v = \frac{U}{2}\sqrt{\frac{\nu}{xU}}\left(\eta f - \int f\,d\eta\right)$$

let $g = \int f\,d\eta \longrightarrow f = g' \longrightarrow f' = g'' \longrightarrow f'' = g'''$

$$*(4) \quad v = \frac{1}{2}\sqrt{\frac{U\nu}{x}}\left(\eta g' - g\right)$$

Sub into Mom Eq

$$-Ug'\left(\frac{\overset{\eta}{\cancel{y}}}{2x}U\sqrt{\frac{U}{x\nu}}\,g''\right) + \frac{1}{2}\sqrt{\frac{U\nu}{x}}\left(\eta g' - g\right)U g''\sqrt{\frac{U}{x\nu}} = \nu\frac{U^2}{x\nu}g'''$$

$$\cancel{y = \eta\sqrt{\frac{x\nu}{U}}}$$

$$-\frac{\cancel{U^2}}{2\cancel{x}}\eta g'' + \frac{1}{2}\cdot\frac{\cancel{U^2}}{\cancel{x}}\left(\eta g' - g\right)g'' = \frac{\cancel{U^2}}{\cancel{x}}g'''$$

$$\cancel{-\eta g' g''} + \cancel{\eta g' g''} - gg'' = 2g'''$$

$$*(6) \quad \boxed{gg'' + 2g''' = 0} \qquad \boxed{g = g(\eta)\ ;}$$

(1) BC: $y=0 \rightarrow u=0$
$\rightarrow \eta=0,\ f=g'=$

(5) BC: $y=0 \rightarrow v=0 \rightarrow \eta=$
$g=0$

(1) BC: $y=\infty \rightarrow u=0$
$\eta=\infty,\ f=g'=1$

Now Solve

$$\begin{cases} gg'' + 2g''' = 0 \end{cases}$$

$\eta = 0 \quad g' = 0$

$\eta = 0 \quad g = 0$

$\eta = \infty \quad g' = 1$

let $h = g' \rightarrow g'' = h', \quad g''' = h''$

$$\begin{cases} gh' + 2h'' = 0 \\ g' = h \end{cases}$$

let $k = h' \rightarrow h'' = k'$

$$\begin{cases} gh' + 2k' = 0 \\ g' = h \\ h' = k \end{cases}$$

or

$$\left.\begin{array}{l} gk + 2k' = 0 \\ g' = h \\ h' = k \end{array}\right\} \rightarrow \begin{bmatrix} k' = -\frac{1}{2} g k \\ h' = k \\ g' = h \end{bmatrix} \quad \begin{array}{ll} \eta = 0, & h = 0 \\ & g = 0 \\ \eta = \infty & h = 1 \end{array}$$

Guess $k(\eta = 0)$

Solve $\rightarrow h(\eta = \infty)$

repeat to converge $h(\eta = \infty) = 1$

$$\boxed{\frac{u}{U} = h(\eta)}$$

Use a shooting method w/ Newton's method.

Converge $F(k_0) = h_\infty - 1 = 0$

(Plug in k_0, Get out $h(\eta = \infty)$; Solve root $F(\cancel{k_\infty}) = 0$)