

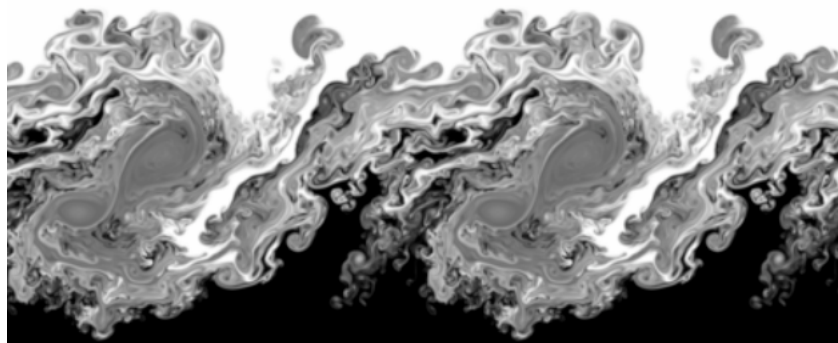
## Turbulence examples

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## Turbulent structures

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## Poem: Siphonaptera

Big fleas have little fleas upon their backs to bite 'em  
And little fleas have lesser fleas, and so, ad infinitum.  
And the great fleas, themselves, in turn, have greater fleas to go on;  
While these again have greater still, and greater still, and so on.

By Mathematicial Agustus De Morgan  
[https://en.wikipedia.org/wiki/Siphonaptera\\_\(poem\)](https://en.wikipedia.org/wiki/Siphonaptera_(poem))



## Richardson (1922)

Big whorls have little whorls  
Which feed on their velocity  
And little whorls have lesser whorls  
And so on to viscosity

[https://en.wikipedia.org/wiki/Siphonaptera\\_\(poem\)](https://en.wikipedia.org/wiki/Siphonaptera_(poem))



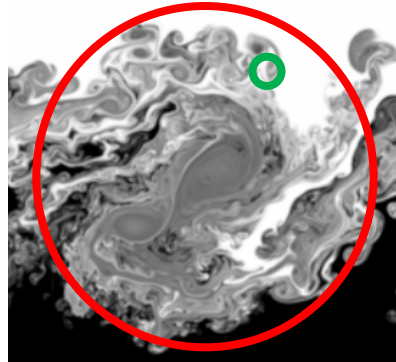
## Turbulent scales

- Turbulence is composed of eddies of different sizes
  - Largest scale: **Integral scale  $L$**
  - Smallest scale: **Kolmogorov scale  $\eta$** .
- Turbulent kinetic energy (tke) is considered.
- Viscosity dissipates tke at the smallest scales
  - But large scales first break down to small scales in the turbulence cascade.

- Energy dissipation rate

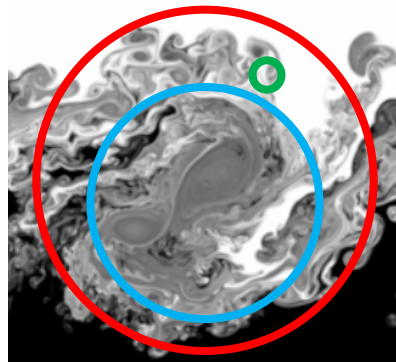
$$\epsilon = \frac{u^2}{\tau} = \frac{u^2}{l/u} = \frac{u^3}{l}$$

- This is independent of the viscosity!



## Turbulent scales

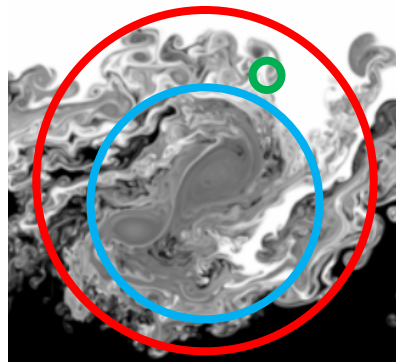
- Kolmogorov's first hypothesis:
  - At sufficiently high Re, the small-scale turbulent motions are statistically isotropic.
  - Hence, universal in form, not affected by the boundary conditions.
  - Intermediate size eddies (**inertial range eddies**) don't depend on viscosity.
  - So they only depend on  $\epsilon$  and  $l$ .
  - In the inertial range,  $\epsilon$  is constant with size  $l$ .
  - This implies a relationship between eddy timescales and lengthscales.



## Turbulent scales

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- Kolmogorov's second hypothesis:
  - At sufficiently high  $Re$ , the statistics of the smallest scale motions have a universal form that is uniquely determined by  $\nu$ ,  $\epsilon$ .
  - Given this, we can form the Kolmogorov scales for length, velocity, and time.



## Turbulent scales

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- Kolmogorov's second hypothesis:
  - At sufficiently high  $Re$ , the statistics of the smallest scale motions have a universal form that is uniquely determined by  $\nu$ ,  $\epsilon$ .
  - Given this, we can form the Kolmogorov scales for length, velocity, and time.

$$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4} \quad u_\eta = (\epsilon\nu)^{1/4} \quad \tau_\eta = (\nu/\epsilon)^{1/2}$$

- Write ratios of Kolmogorov to Integral scales in terms of  $Re$

$$\epsilon = \frac{u^3}{l}$$

$$Re = \frac{uL}{\nu}$$

$$\frac{L}{\eta} = Re^{3/4}$$

$$\frac{u}{u_\eta} = Re^{1/4}$$

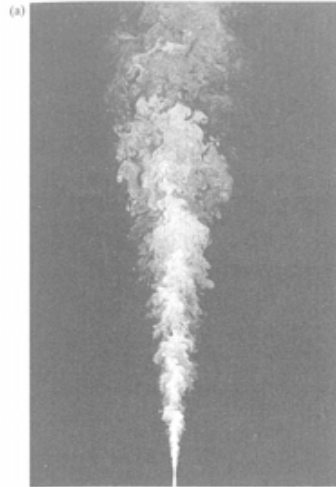
$$\frac{\tau}{\tau_\eta} = Re^{1/2}$$



# Turbulent Jets

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Re=5000



Re=20000

