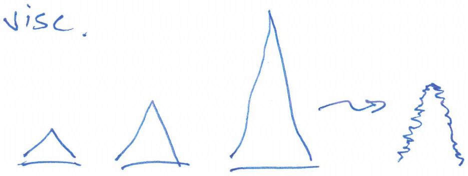
Turbulence.

- Characteristics.
- Scaling
- Basic modeling.
- Turbulent jets.

- Most practical Comb. is Turbulent.
  - large Size
  - high velocity $\longrightarrow$ high Re.
- Desireable for mixing.
- Nonreacting jets $\longrightarrow$ jet flames.

- Characteristics.
  - Why Does turb. happen?
    - Reynolds experiment: pipe movie.
    - $Re \sim \frac{F_{mom}}{F_{visc}}$; Flow instabilities are not damped by Viscosity.
      - 2 key forces: mom, visc.
    - Jet w/ Inc. Velocity (Re)



      - As V_e inc., Gradients (where visc. Dissipates the exit momentum) increases, and Gets very large.
      - Diss. of mom $\sim \tau \cdot A = \mu \frac{du}{dx} \cdot A$
      - Large area is not Stable $\longrightarrow$ breaks apart
        - Inc. S.A.
        - Dec. (stable) Gradients

        } = turbulence.

- Kelvin-Helmholz movie.
  - observations ??
    1. Large eddies engulf fluid
    2. Small eddies grow to big eddies that Decompose to small eddies
       Turbulent Cascade
    3. Large S.A. (interface)
    4. Mixing rate

Richardson 1922
Poem.

Scaling.

Kolmogorov theorems.

Turb. consists of eddies of Different sizes.

- Consider Scales of turbulence.
  - Integral Scale : large eddies : L, u $\rightarrow$ $\tau = L/u$
  - Kolmogorov Scale : Small eddies : η, u_η $\rightarrow$ $\tau_\eta = \eta/u_\eta$.
  - $Re = \dfrac{Lu}{\nu}$
  - $Re_\eta = \dfrac{\eta u_\eta}{\nu}$

- See Slides

$$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4}$$

$$u_\eta = (\epsilon\nu)^{1/4}$$

$$t_\eta = (\nu/\epsilon)^{1/2}$$

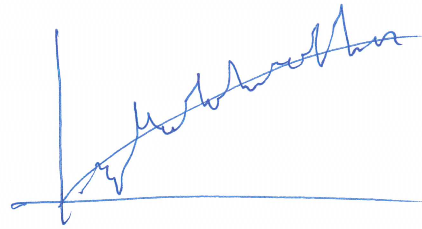
$$\xrightarrow[\;]{\epsilon = u^3/l,\ Re = Lu/\nu}$$

$$\frac{L}{\eta} = Re^{3/4}$$

$$\frac{u}{u_\eta} = Re^{1/4}$$

$$\frac{\tau}{\tau_\eta} = Re^{1/2}$$

Basic modeling



Reynold Decomposition.

$$u = \bar{u} + u' \qquad \bar{u} = \frac{1}{\Delta t}\int_0^{\Delta t} u(t)\,dt$$

$$\overline{u'} = 0$$

- Resolving all scales is <u>expensive</u>

  $\sim Re^3$

- Solve average flow field. → Reynold's Decomp.
- → Need eqns.
- Apply averaging to N.S.

$$\nabla\cdot\vec{V} = 0$$

$$\frac{\partial\vec{V}}{\partial t} + \nabla\cdot\vec{V}\vec{V} = -\frac{1}{\rho}\nabla P - \frac{1}{\rho}\nabla\cdot\tau + \vec{g}$$

2-D BL eq

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}uu + \frac{\partial}{\partial y}uv = \nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial}{\partial x}(\bar{u}+u')(\bar{u}+u') = \frac{\partial}{\partial x}(\bar{u}\bar{u} + 2\bar{u}u' + u'u')$$

Avg: $$\frac{\partial}{\partial x}\left(\overline{\bar{u}\bar{u} + 2\bar{u}u' + u'u'}\right)$$

$$= \frac{\partial}{\partial x}\left(\bar{u}\bar{u} + \cancel{2\bar{u}\overline{u'}} + \overline{u'u'}\right)$$

other terms too.

$$\rightarrow \overset{S.S.}{\cancel{\frac{\partial\bar{u}}{\partial t}}} + \frac{\partial}{\partial x}\bar{u}\bar{u} + \frac{\partial}{\partial y}\bar{u}\bar{v} = \nu\frac{\partial^2\bar{u}}{\partial y^2} - \overset{\text{ignore}}{\cancel{\frac{\partial}{\partial x}\overline{u'u'}}} - \frac{\partial}{\partial y}\overline{u'v'}$$

- Same eqn, w/ overbar AND New term

New term is $-\frac{\partial}{\partial y}\overline{u'v'}$

⊛ $\overline{u'v'}$ is a viscous stress.

$$\frac{\tau}{\rho} = \frac{\mu}{\rho}\frac{\partial u}{dx} \quad (=) \quad \frac{m^2}{s}\cdot\frac{m}{s}\cdot\frac{1}{m} = \frac{m^2}{s^2}$$

$$\overline{u'v'} \quad (=) \quad \frac{m^2}{s^2}$$

Model as a $\overline{u'v'} \sim \nu_{turb}\cdot\frac{\partial \bar{u}}{\partial y}$

$$\nu_{turb} \; (=) \; \frac{m^2}{s} \rightarrow \frac{l_m^2}{T} = l_m^2 \left|\frac{d\bar{u}}{dy}\right|$$

$$\simeq 0.1365\, l_m(\bar{V}_{max} - \bar{V}_{min})$$

$l_m = 0.075\, \delta_{99\%}$ for jets

Jet soln.

Eqns same as laminar, but w/ μ_{turb} instead of μ

→ Same solution as laminar.

But the form of μ_{turb} gives diff. behavior.

Turb.	Laminar.
$\frac{\bar{u}_0}{\bar{u}_e} = 13.15\,\frac{R}{x}$	$\frac{u_0}{u_e} = 0.375\,Re\cdot\frac{R}{x}$
$\frac{r_{1/2}}{x} = 0.08468$	$\frac{r_{1/2}}{x} = 2.97/Re.$
indep. of Re!	Dep. on Re.!