

## Radiant fraction.

See Turns plots. (slides)

$$X_R = \frac{\dot{Q}_{\text{rad}}}{\dot{m} \Delta h_c} = \frac{k \cdot \nu \cdot \sigma T_f^4}{\rho \nu_e A \Delta h_c} \propto \frac{k d^3 T^4}{\nu d^2} \propto \frac{k T^4 d}{\nu} \propto k T^4 \tau$$

$k$  is absorption coeff

$T$  is temperature

$\tau$  is residence time

Turbulent flames  $\rightarrow$  constant size / shape as flow increases for given diameter jet exit.

Higher flow  $\rightarrow$  shorter  $\tau \rightarrow$  lower  $X_R$   
for given  $d$

Larger  $d_j \rightarrow$  larger flame  $\rightarrow$  larger  $\tau \rightarrow$  larger  $X_R$   
for given flow

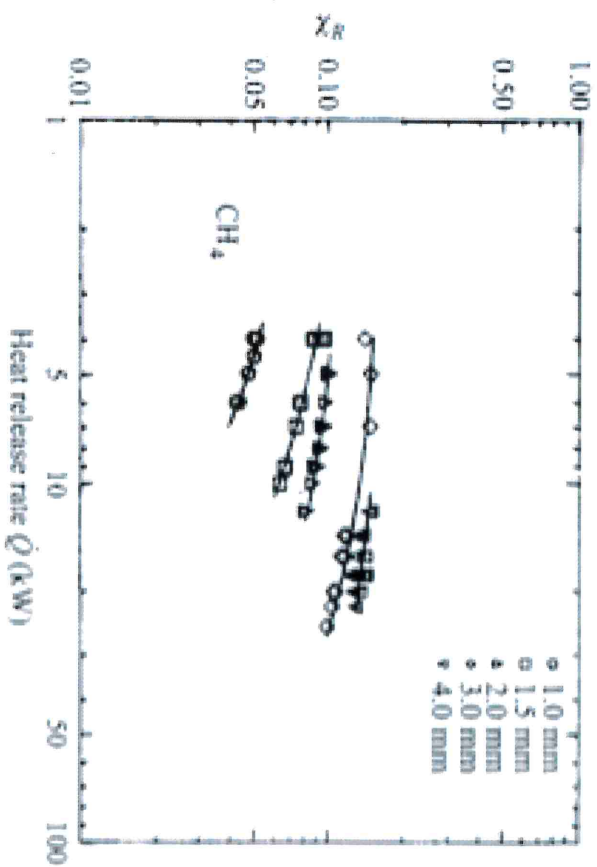
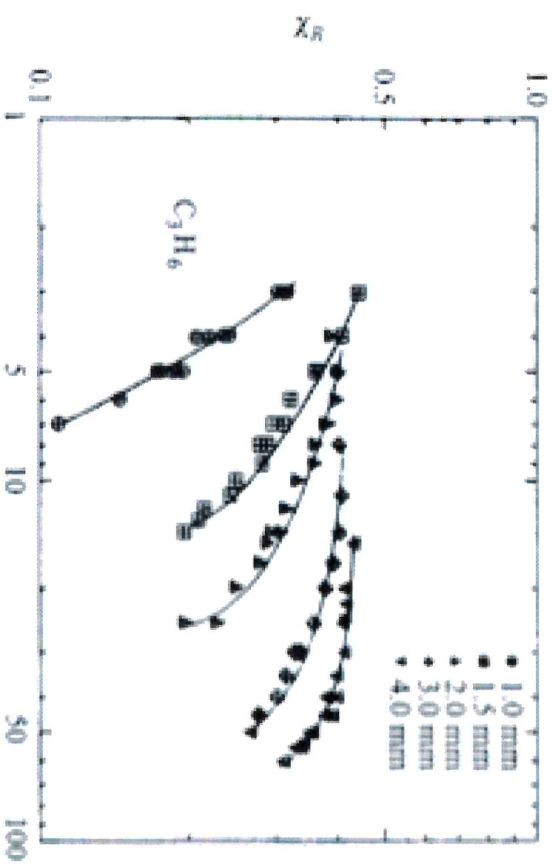
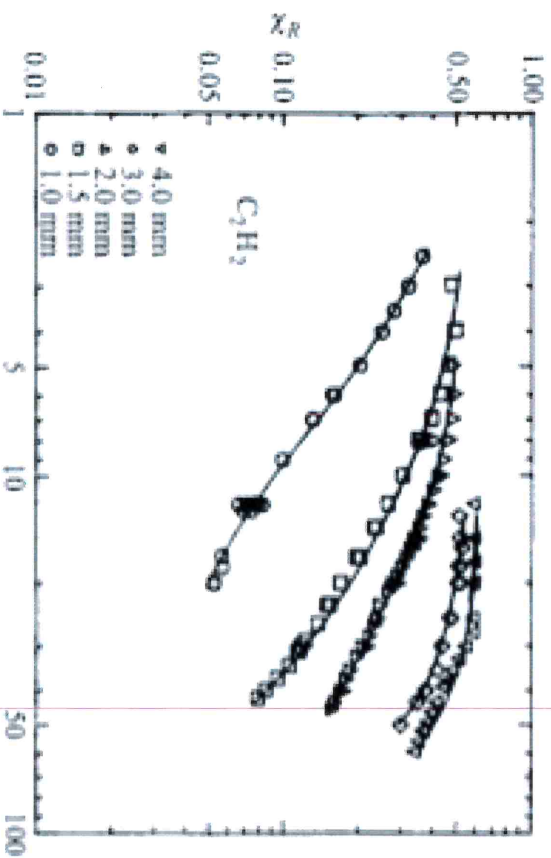
Shorter  $\tau \rightarrow$  less soot, higher strain  $\rightarrow$  lower  $T$ , both of which reduce radiation.

Loss fractions:

$C_2H_2$	$X_{R, \text{max}} = 60\%$
$C_3H_6$	$= 45\%$
$CH_4$	$= 15\%$

$\rightarrow$  fuel / sooting tendency.

# Radiation

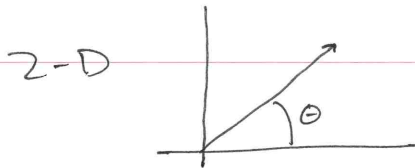


# Intensity

I is Directional

$$I(\theta) = \frac{W}{m^2 \cdot \lambda \cdot \text{St}}$$

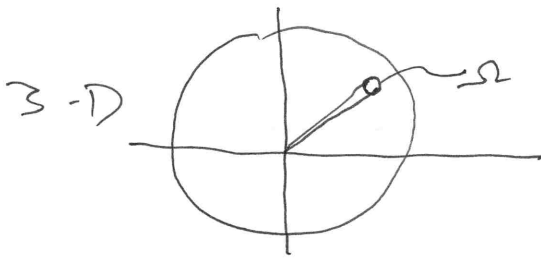
I is energy per time per unit area perpendicular to the rays ( $m^2$ )  
 Per unit wavelength of the light ( $\lambda$ )  
 Per unit Steradian (St), or, rather, per unit Solid angle.



Angle:  $0 - 2\pi$

$2\pi$  radians in a Circle.

$$C = 2\pi r = \int_0^{2\pi} r d\theta$$



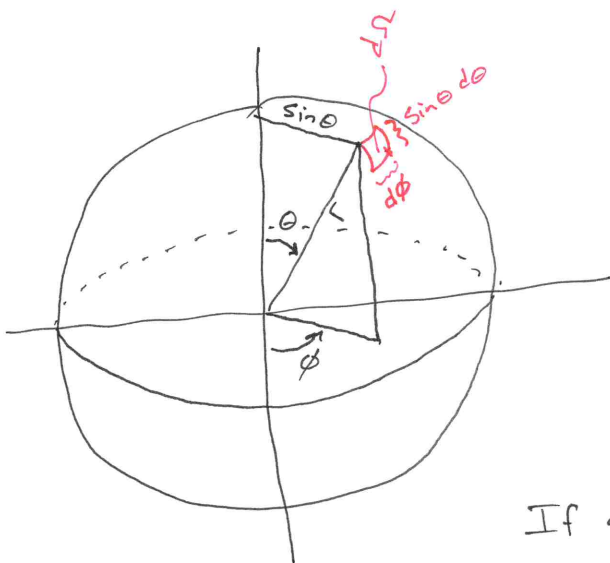
Solid angle:  $0 - 4\pi$

$4\pi$  steradians in a Sphere.

$$A = 4\pi r^2 = \int_0^{4\pi} r^2 d\Omega$$

$$= \int_0^{2\pi} \int_0^{\pi} r^2 \sin\theta d\theta d\phi$$

where  $d\Omega = \sin\theta d\theta d\phi$



Patch  $d\Omega = (\sin\theta d\theta) \cdot (d\phi)$

If all stars were the same as our Sun then we would see the same intensity looking at a star as we would looking at the Sun.

But our eye receives more energy because energy is  $I \cdot \Delta\Omega$  so the Sun has more  $\Delta\Omega$  than a star.  
 $\cdot A$

# Heat flux.

Intensity  $\rightarrow$  Heat flux to a Surface,

- this could be a fictitious Surface.

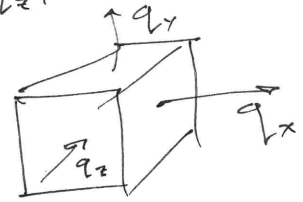
Such as  $\vec{q}_{rad} = q_x \vec{i} + q_y \vec{j} + q_z \vec{k}$

$$\dot{Q}_{rad} = -\nabla \cdot \vec{q}_{rad}$$

$$(\text{=}) \text{ W/m}^3$$

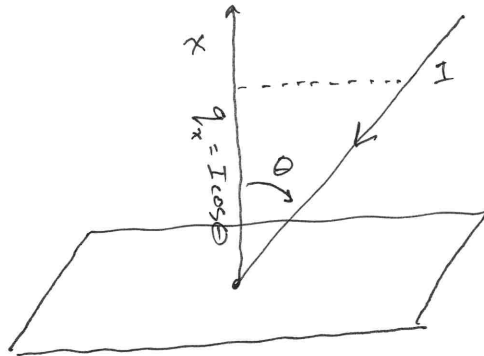
is an energy eqn source term.

$$\frac{\partial \rho h}{\partial t} + \nabla \cdot (\rho \vec{v} h) + \nabla \cdot \vec{q} = -\nabla \cdot \vec{q}_{rad} = \dot{Q}_{rad}$$



- Get  $q_x, q_y, q_z$  from an intensity field.

where  $q_x$  (etc) is like the net heat flux on the X-Surface.

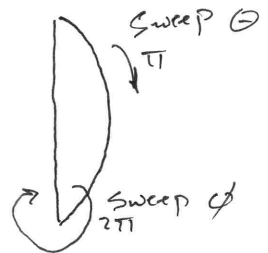


$q_x = I \cos \theta$  is the Projection of  $I$  onto Surface  $x$ .

$$q_r = \int_{4\pi} I \cos \theta d\Omega$$

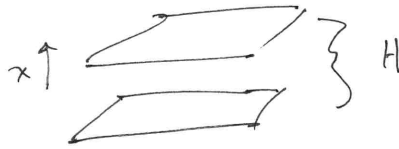
$$q = \int_0^{2\pi} \int_0^\pi \underbrace{I \cos \theta \sin \theta d\theta d\phi}_{d\Omega}$$

•  $\int$  Sphere:



## Example

- $\infty$  Parallel plates.



- $\vec{q} = q_x$  since  $q_y = q_z = 0$

- 1-Dimensional

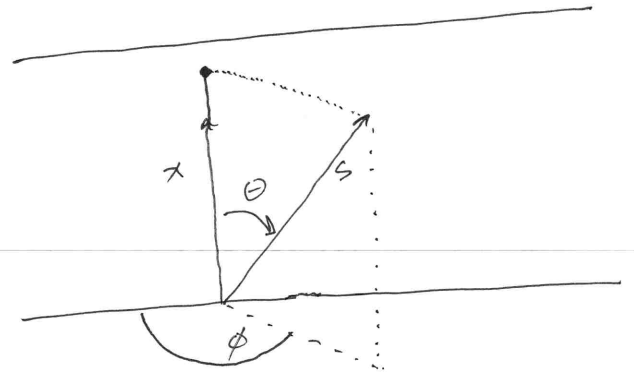
- Assume uniform properties of a gas between the plates  $\rightarrow$  const  $k$

$$\frac{dI}{ds} = kI_b - kI$$

$$I_b = \frac{E_b}{\pi} = \frac{\sigma T_g^4}{\pi}$$

$$I(s=0) = I_0 = \frac{\sigma T_w^4}{\pi}$$

H



in a given  $\Delta s$ ,  $I$  changes due to emission ( $kI_b$ ), which increases  $I$ , and absorption ( $-kI$ ) which decreases  $I$ .

- $s$  is path length along direction of  $I$

$$\bullet x = s \cos \theta$$

$$\bullet ds = dx / \cos \theta$$

Absorption decreases  $I$  in proportion

to  $I$ , where  $k (=) \frac{1}{m}$  is the

Proportionality constant (absorption coefficient)

Note  $(1/k) (=) m$  is a characteristic length scale for radiation.

This is the nominal optical thickness of a gas.

$(x \cdot k)$  is a non-dimensional:  $(xk) \ll 1$  is optically thin and radiation passes right through.

$(xk) \gg 1$  is optically thick

$$\frac{dI}{ds} = kI_b - kI$$

Analytic Solution for const  $k$ : (Separate & integrate)

$$I = I_b - (I_b - I_0) e^{-ks}$$

$$(*) \quad I = I_b - (I_b - I_0) e^{-kx/\cos\theta}$$

$$q = \int_0^{2\pi} \int_0^\pi I \cos\theta \sin\theta d\theta d\phi$$

Due to symmetry with respect to  $\phi$

$$(*) \quad q = 2\pi \int_0^\pi I \cos\theta \sin\theta d\theta$$

Insert  $I$ :

$$q = 2\pi \int_0^\pi \cos\theta \sin\theta \left[ I_b - (I_b - I_0) e^{-kx/\cos\theta} \right] d\theta$$

- here,  $x$  is the distance from the boundary with  $I = I_0$ .
- the geometry of our problem, then, suggests splitting into 2 hemispheres:

$$(*) \quad q = 2\pi \int_0^{\pi/2} \cos\theta \sin\theta \left[ I_b - (I_b - I_0) e^{-kx/\cos\theta} \right] d\theta - 2\pi \int_0^{\pi/2} \cos\theta \sin\theta \left[ I_b - (I_b - I_0) e^{-k(H-x)/\cos\theta} \right] d\theta$$

- The first term is  $q$  arising from  $I$  with positive  $x$  component
  - The second term is  $q$  arising from  $I$  with negative  $x$  component,
- ↗ upward  
↘ downward

At the wall,  $x = 0$ :

$$(*) \quad q_w = 2\pi \int_0^{\pi/2} I_0 \cos\theta \sin\theta d\theta - 2\pi \int_0^{\pi/2} \cos\theta \sin\theta \left[ I_b - (I_b - I_0) e^{-kH/\cos\theta} \right] d\theta$$

- $q_w$  will be positive if heat is leaving the wall
- $q_w$  " " negative " " " entering the wall.

$$Q = - \frac{dq}{dx}$$

$$Q(=) \text{ W/m}^3 = -\nabla \cdot q$$

$Q$  is positive for local radiative addition  $\rightarrow T$  will increase.

$\nabla \cdot q$  is net "flux" out per volume

$-\nabla \cdot q$  then is net "flux" in per unit Vol.

$$\begin{aligned} (*) \quad Q &= -2\pi \int_0^{\pi/2} k \sin \theta (\bar{I}_b - \bar{I}_0) e^{+kx/\cos \theta} d\theta \\ &\quad - 2\pi \int_0^{\pi/2} k \sin \theta (\bar{I}_b - \bar{I}_0) e^{-k(H-x)/\cos \theta} d\theta \end{aligned}$$

$$(*) \quad Q = -2\pi k (\bar{I}_b - \bar{I}_0) \int_0^{\pi/2} \sin \theta \left( e^{-kx/\cos \theta} + e^{-k(H-x)/\cos \theta} \right) d\theta$$

## Absorption Coefficient.

A simple gray (meaning non-spectral) approximation for  $k$  is the Planck-Mean assumption.

$$k_{PM} = \frac{\pi}{\sigma T^4} \int_0^{\infty} I_{b,\eta} k_{\eta} d\eta \quad \text{where } \eta \text{ is the spectral wavenumber.}$$

$k_{PM}$  is normally correlated to  $T$  as a polynomial.

for a given species, we have

$$k_{PM,i} = \chi_i P \cdot a(T)$$

where  $\chi_i$  is mole fraction

$P$  is pressure

$a(T)$  is a polynomial

$$k_{PM} (\text{W/m}^2 \cdot \text{K})$$

See, for example [www.sandia.gov/TNF/radiation.htm](http://www.sandia.gov/TNF/radiation.htm).

For soot:

$$k = 1817 f_v \cdot T \quad \text{where } f_v \text{ is volume fraction}$$

$T$  is Kelvin

(Williams & Shaddix *IJHMT* 50 P1616-1630, 2007)

(In Modest 3<sup>rd</sup> Edition use 1477 for Chang)

$$k_{Mix} = \sum k_i$$



# Planck Mean Absorption Coefficient

Modest, Radiative Heat Transfer, 3<sup>rd</sup> Edition, p 369.

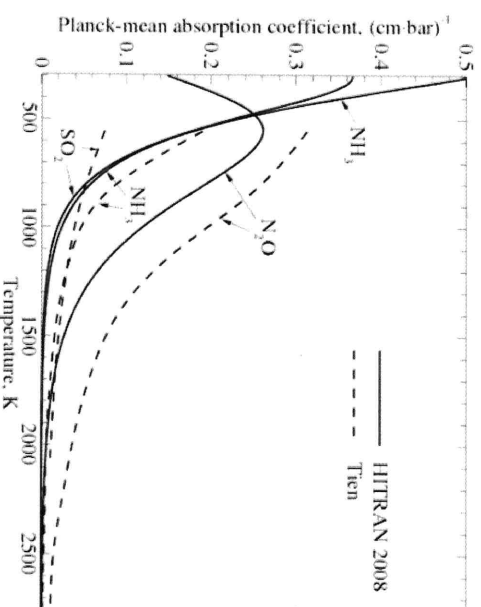
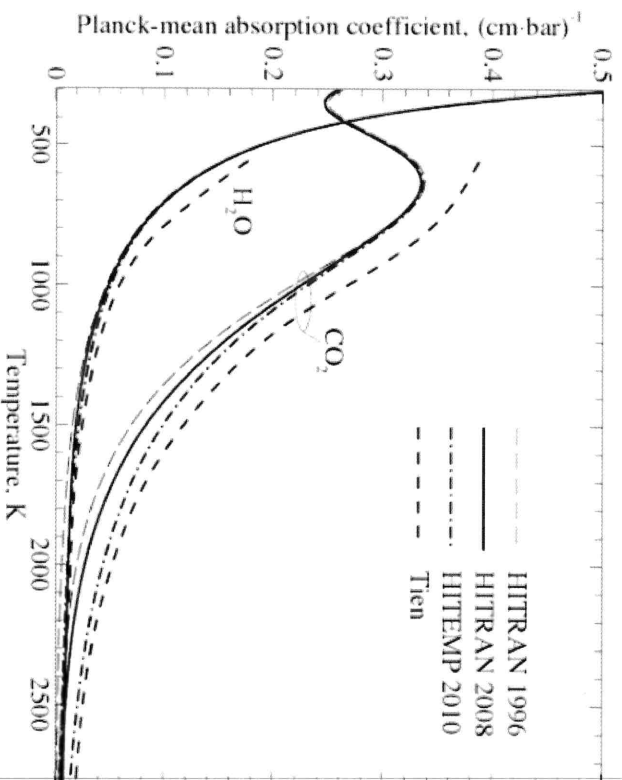


FIGURE 11-32 Planck-mean absorption coefficients for ammonia, nitrous oxide, and sulfur dioxide.

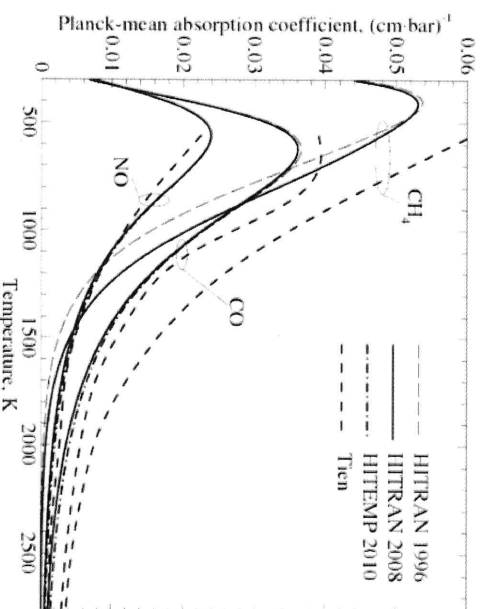


FIGURE 11-33 Planck-mean absorption coefficients for carbon monoxide, nitric oxide, and methane.

FIGURE 11-31 Planck-mean absorption coefficients for carbon dioxide, and water vapor.