# Class 24 - Premixed flames - Analysis

Flame Analysis.

- Simplified
- T, P Dep.
- (Detailed)

## Simplified Analysis.

Many theories → 3 Groups

1. Thermal Theory
   - Divide the flame into 2 zones
     - Preheat Zone
     - Rxn zone,
   - Flame Propagates Due to Heat Transfer / Generation
2. Diffusion theory.
   - Diffusion of radicals, not heat is controlling
3. Comprehensive Theory
   - Like Thermal, but also include Species with $Le = 1$

⊙ Turns gives a Thermal theory by Spaulding

⊙ Theory of Mallard & Le Chatlier is Somewhat more General, but reduces to the Same thing under Spauldings Assumptions (See Kuo's Book)

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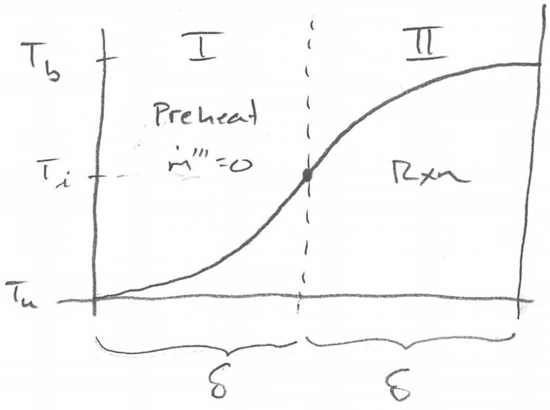
Flame Propagates as Reactions increase T → a T Gradient → Heat flux to unburnt mixture, which increases in T to $T_i$, an ignition T

Split flames into 2 Zones, A preheat zone I, Reaction zone II

# Simplified Analysis of Flame Speed, thickness

Flame:



Note: Turns Diff. By factor of 2 Due to Def. of δ, $\dot m_F'''$

$\boxed{S_L}$

## Shvab - Zeldovich

$$\dot m'' c_p \frac{dT}{dx} = \frac{d}{dx}\left(\lambda \frac{dT}{dx}\right) - \sum_i \dot m_i''' h_{fi}^\circ$$

### Zone I

$$\dot m'' c_p \frac{dT}{dx} = \lambda \frac{d^2T}{dx^2} - 0$$

$c_p, \lambda \approx$ const.

Scale: $\frac{dT}{dx} \sim \frac{\Delta T}{\delta}$ , $\frac{d^2T}{dx^2} \sim \frac{\Delta T}{\delta^2}$

$$\dot m'' c_p \frac{\cancel{\Delta T}}{\cancel{\delta}} = \lambda \frac{\cancel{\Delta T}}{\delta^{\cancel{2}}} \rightarrow \dot m'' = \frac{\lambda}{c_p \delta}$$

$$\dot m'' = \rho_u S_L$$

$$\rightarrow S_L = \frac{\lambda}{\rho_u c_p \delta} = \boxed{\frac{\alpha}{\delta} = S_L} \quad ①$$

$\boxed{\delta}$

Zone II ; $\dot m_F'' \cancel{A} = \dot m_F''' \cdot (\cancel{A} \cdot \delta)$

$$\underset{kg}{(1)}(F) + \underset{kg}{(\nu)}(\text{air}) \rightarrow (1+\nu)(\text{Prod})$$

$$\dot m_F'' = \dot m''(Y_F) = \rho_u S_L\left(\frac{1}{1+\nu}\right) \quad ②$$

$$\rho_u S_L\left(\frac{1}{1+\nu}\right) = \dot m_F''' \cdot \delta \rightarrow \boxed{\delta = \frac{\rho_u S_L}{\dot m_F'''(1+\nu)}}$$

$$\rightarrow S_L = \sqrt{\frac{\alpha \dot m_F'''(1+\nu)}{\rho_u}}$$

(by insert ② into ①)

## Key Result

$$S_L \propto \sqrt{\frac{\alpha \dot{m}'''}{\rho_u}}$$

$$\delta \propto \frac{\alpha}{S_L}$$

## T, P Dep - See Turns

$\alpha \propto \frac{T_u \bar{T}^{0.75}}{P}$ $\qquad \alpha = \frac{\lambda}{\rho c_p}$

$\rho_u \propto \frac{P}{T_u}$

$\dot{m}''' \propto T^{-n} P^n e^{-E/RT}$ $\qquad \dot{m}''' = A e^{-E/RT} [\ ]^n \ ; \ [\ ] = \frac{P}{RT}$

T: $S_L \propto T_u \bar{T}^{0.375} T_b^{-n/2} \exp(-E/2RT_b)$

$\delta \propto \bar{T}^{0.375} T_b^{n/2} \exp(E/2RT_b)$

P: $S_L \propto P^{(n-2)/2}$

$\delta \propto P^{-n/2}$

for $n \approx 2$ $\quad S_L \neq S_L(P)$

both $S_L$, $\delta$ are weak functions

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See PPT Slides with Plots,

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## Flame Speed Correlations

(8.33) - Turns.

$$S_L = S_{L,ref}\left(\frac{T_u}{T_{u,r}}\right)^{\gamma}\left(\frac{P}{P_r}\right)^{\beta}(1-2.1\,Y_{dil})$$

$\gamma = 2.18 - 0.8(\phi - 1)$

$\beta = -0.16 + 0.22(\phi - 1)$ $\qquad$ ref = 1 atm, 298 K

$S_{L,ref} = B_M + B_2(\phi - \phi_M)^2$ $\qquad B_M, B_2, \phi_M \rightarrow$ Table 8.3

# Temperature Dependence

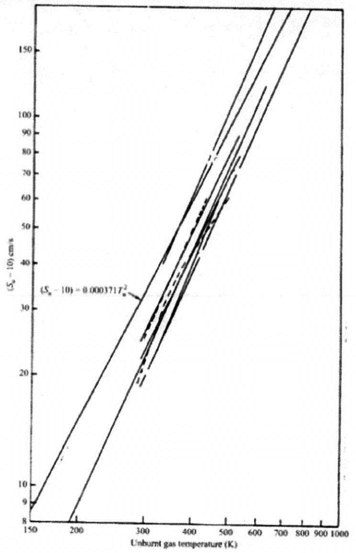

**Figure 8.13** Effect of unburned gas temperature on laminar flame speeds of stoichiometric methane-air mixtures at 1 atm. Various lines are data from various investigators.

$$S_L \propto \bar{T}^{0.375} T_u T_b^{-n/2} \exp(-E/2RT_b)$$

Experimental for methan → $T_u^2$

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# Pressure Dependence

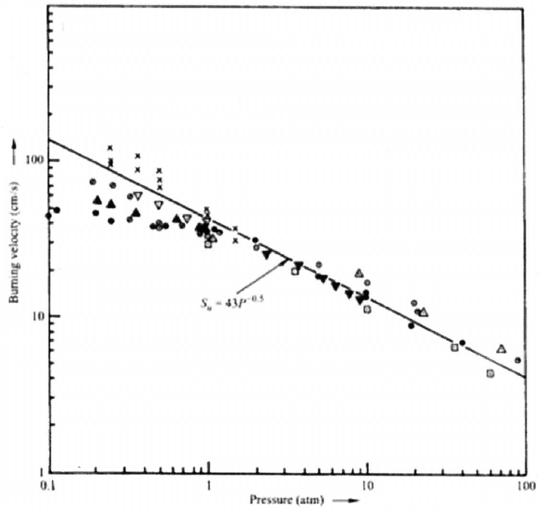

**Figure 8.14** Effect of pressure on laminar flame speeds of stoichiometric methane–air mixtures for $T_u$ = 16–27 °C.

Simplified Analysis says:

$$S_L \propto P^{(n-2)/2}$$

For n=2 this gives no dependence.

Methane, n=1 (Table 5.1) → n=-0.5, consistent with this plot

# Equivalence Ratio

Flame speed with equivalence ratio is a temperature effect

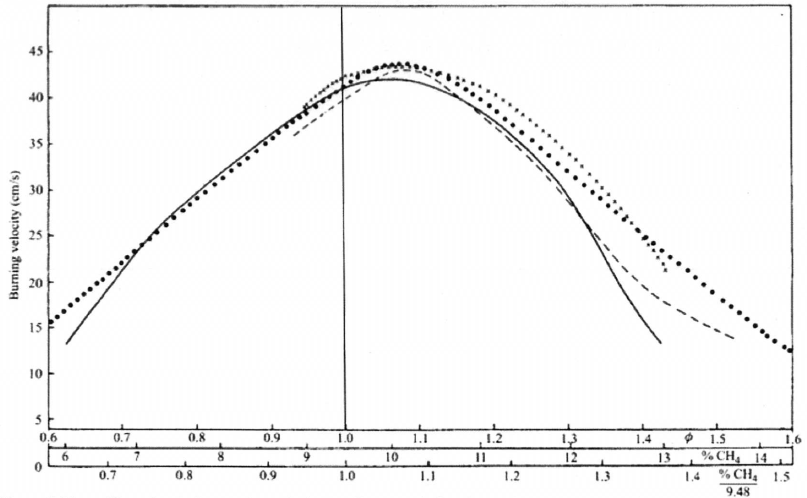

**Figure 8.15** Effect of equivalence ratio on the laminar flame speed of methane–air mixtures at atmospheric pressure.
| SOURCE: Reprinted with permission, Elsevier Science, Inc., from Ref. [19], © 1972, The Combustion Institute.

# Methane Flame Thickness

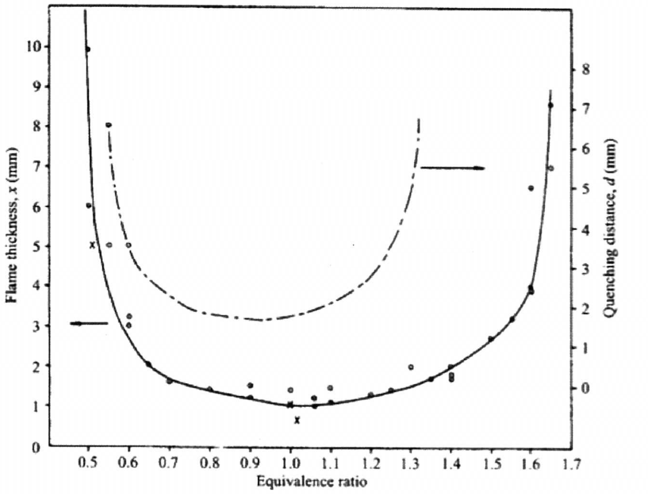

**Figure 8.16** Flame thickness for laminar methane–air flames at atmospheric pressure. Also shown is the quenching distance.

# Speed for Several Fuels

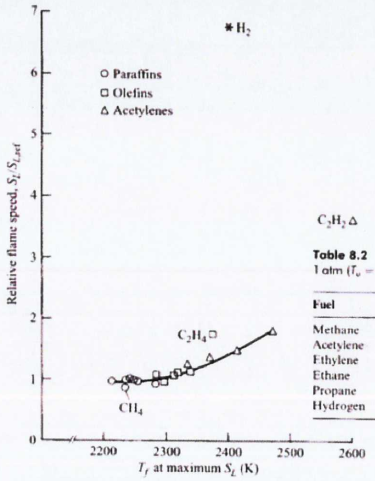

**Table 8.2** Laminar flame speeds for various pure fuels burning in air for $\Phi = 1.0$ and at 1 atm ($T_u$ = room temperature) from Ref. [20]

| Fuel | Formula | Laminar Flame Speeds, $S_L$ (cm/s) |
|---|---|---|
| Methane | $CH_4$ | 40 |
| Acetylene | $C_2H_2$ | 136 |
| Ethylene | $C_2H_4$ | 67 |
| Ethane | $C_2H_6$ | 43 |
| Propane | $C_3H_8$ | 44 |
| Hydrogen | $H_2$ | 210 |

**Figure 8.17** Relative flame speeds for $C_1$–$C_6$ hydrocarbon fuels. The reference flame speed is based on propane using the tube method [21].

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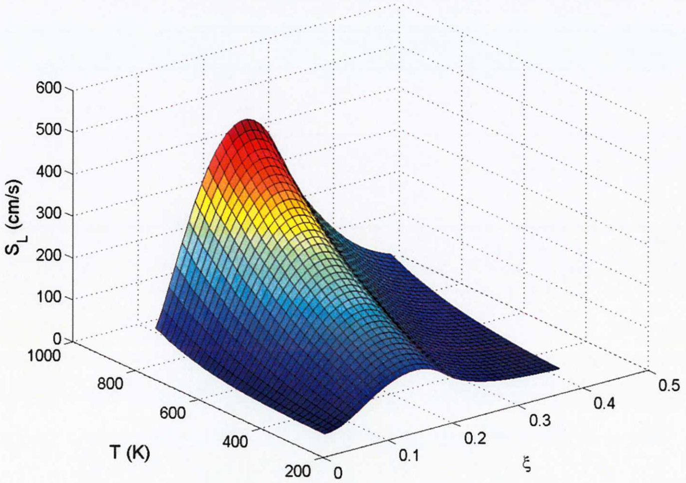
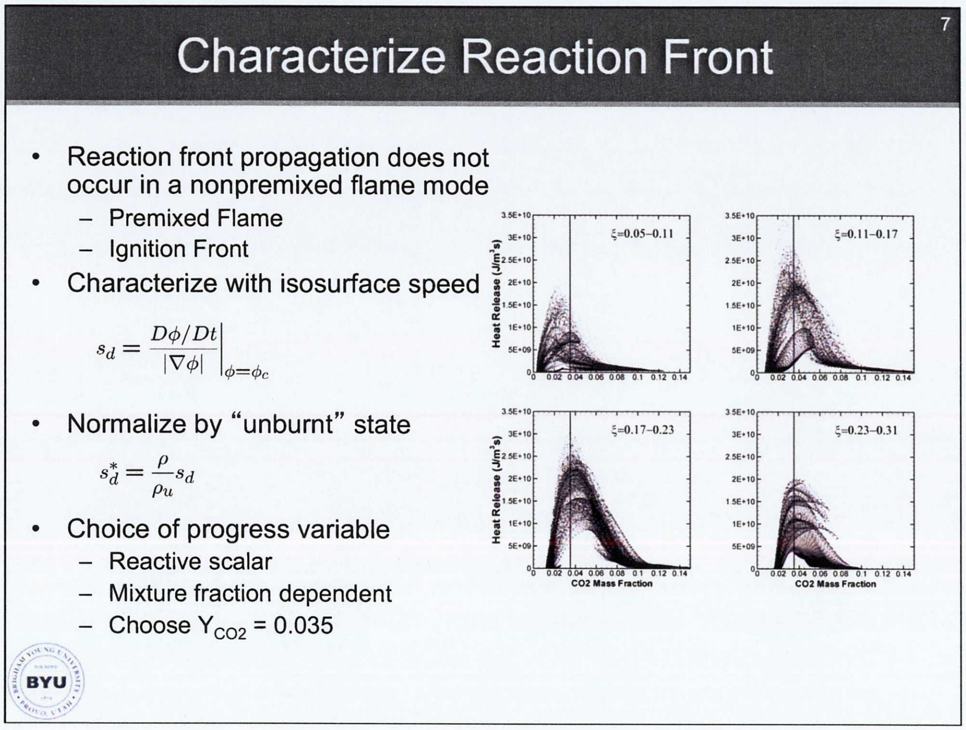
# Ethylene Speed

Run with the Chemkin Premix Code. Each point is a separate Run





# Characterize Reaction Front

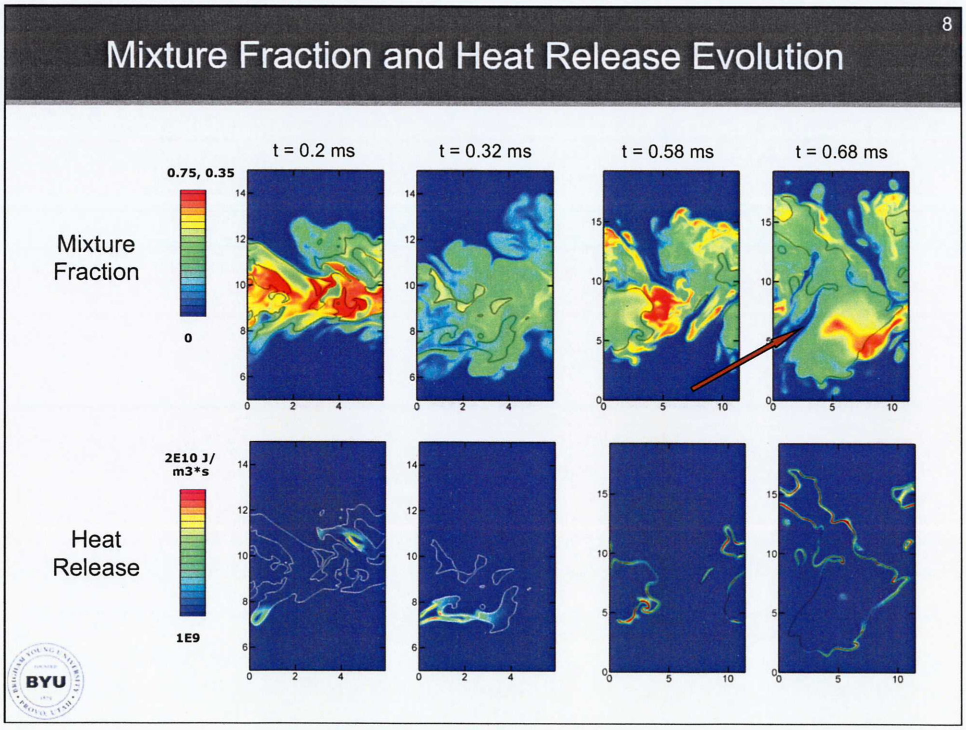
- Reaction front propagation does not occur in a nonpremixed flame mode
  - Premixed Flame
  - Ignition Front
- Characterize with isosurface speed

$$s_d = \left. \frac{D\phi/Dt}{|\nabla\phi|} \right|_{\phi=\phi_c}$$

- Normalize by "unburnt" state

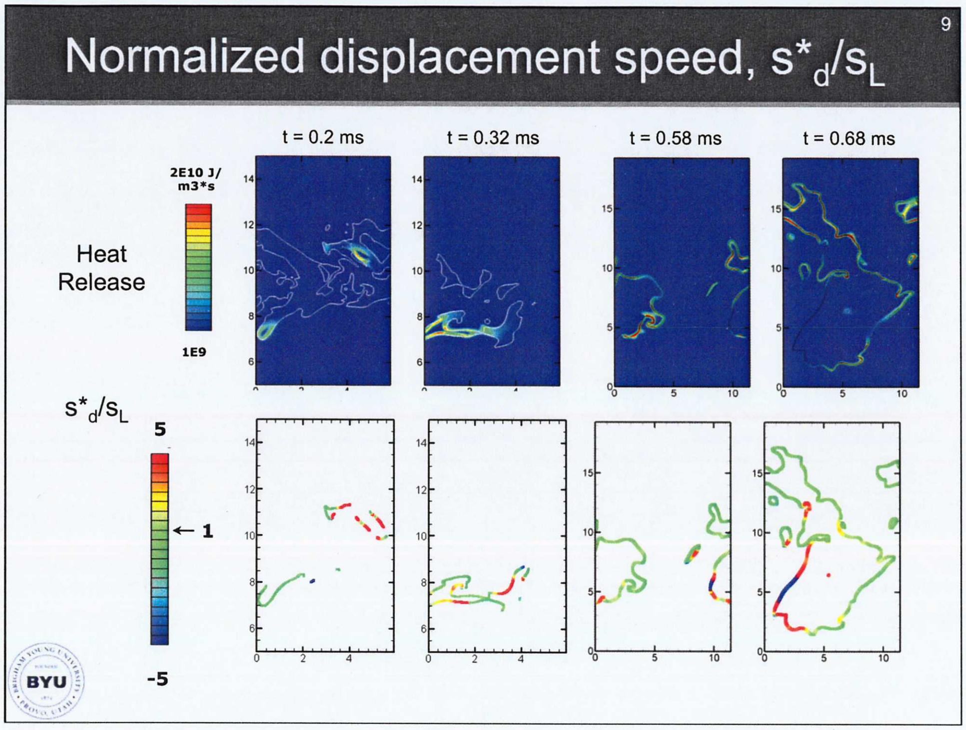
$$s_d^* = \frac{\rho}{\rho_u} s_d$$

- Choice of progress variable
  - Reactive scalar
  - Mixture fraction dependent
  - Choose $Y_{CO2} = 0.035$



# Mixture Fraction and Heat Release Evolution

# Normalized displacement speed, $s^*_d/s_L$



# Normalized displacement speed, $s^*_d/s_L$