Lecture 37 – Sprays

- Chapter 10

- Combustion of Liquid Droplets and Sprays is an important area of nonpremixed Combustion.

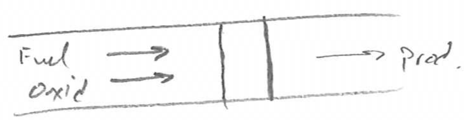
  - Diesel Engines
  - Gas Turbine Engines
  - Rockets

- Process
  - Spray of Droplets injected into hot gas (burning or hot enough to ignite)
  - These Droplets evaporate
  - Burn in gas phase.
  - Rate of Combustion is Determined by the rate of vaporization
    - Fast Chemistry as usual
  - Vaporization rate Determines Droplet lifetime, Hence length of burning zone / time of burning.

- Key Quantities
  - Evaporation Rate
  - Droplet Lifetime.

  } Single particle → consider collection = Spray for a Complete application.

- Example Application.
  - Consider a 1-D (Plug flow) Combustor.
    - "Jet flame"
    - Rocket.



~~Mod~~

→

- Model as 1-D
- Gas phase + Liquid
- 1 Drop Size
- Gas is in equilibrium.

Find: T profile
Y_i profile
d_i profile

Mass: $\frac{d\dot{m}_g}{dx} + \frac{d\dot{m}_l}{dx} = 0$

$\frac{d\dot{m}_l}{dx} = -\dot{m}_{vap}$

Energy: $\frac{d(\dot{m}_g h_g)}{dx} + \frac{d(\dot{m}_l h_l)}{dx} = 0$

$\rightarrow h_g = f(T_g, P_g, \phi_g)$

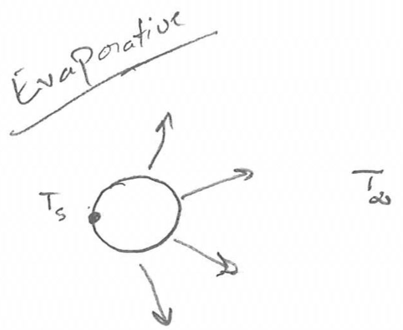
or $T_g = f'(h_g, P_g, \phi_g)$

Evaporation rate
- which Deps on Local T, Comp, velocity.

$\Leftarrow \left\{ \begin{array}{l} \text{Mass, Energy} \rightarrow \phi, h_g \\ \text{Equilibrium} \rightarrow T, Y_i \end{array} \right.$

See page 399 for an example.

Droplet Evaporation: Real vs Ideal



- Convection / Buoyant
- Unsteady
  - Vary size
  - Vary T in Drop
  - Vary T_{surf}
- Multicomponent fuel / vapor
- Our usual complex combustion.
- Transport w/ Variable properties.

Assume
- Spherical
- Quiescent
- Unity Le
- Quasi-steady
- Unif $T_{droplet}$
- $T_{surf} = T_{boil}$
- 1 Fuel component
- Const. Props

Then
- No Mass Transfer (boiling → all fuel)
- Energy Balance → heat of vaporization balanced by conduction to surface,

$$\boxed{\dot{m} h_{vap} = (4\pi r^2)\lambda \left.\frac{dT}{dr}\right|_{surf.}}$$

Need $\left.\frac{dT}{dr}\right|_{surf} \rightarrow$ Get $T(r)$ in gas:

- $\dot{m} = const$: $\frac{d\rho v r^2}{dr} = 0 \rightarrow \rho v r^2 = const$; $\underline{\dot{m} = 4\pi \rho v r^2}$

- (7.65): $\frac{1}{r^2}\frac{d}{dr}\left[r^2 \rho v C_p dT - r^2 \lambda \frac{dT}{dr}\right] = 0$

$$\rightarrow \frac{\dot{m} C_p}{4\pi\lambda}\frac{dT}{dr} = \frac{d}{dr}\left(r^2 \frac{dT}{dr}\right)$$

- Separate variables and integrate twice w/ $T(r=0) = T_s$, $T(r=\infty) = T_\infty$
- evaluate $\left.\frac{dT}{dr}\right|_s$

$$* \quad \dot{m} = \frac{4\pi\lambda r_s}{C_p} \ln\Big[\underbrace{\frac{C_p(T_\infty - T_s)}{\Delta h_{vap}}}_{B_q} + 1\Big] \rightarrow \dot{m} = \frac{4\pi\lambda r_s}{C_p}\ln(B_q + 1)$$

$$\frac{dm}{dt} = -\dot{m} \; ; \quad m = \frac{\pi}{6} D^3 \rho_l \rightarrow \frac{dD^2}{dt} = \frac{-8\lambda}{\rho_l C_{p,g}} \ln(B_q + 1) = -K$$

- Separate vars and integrate.

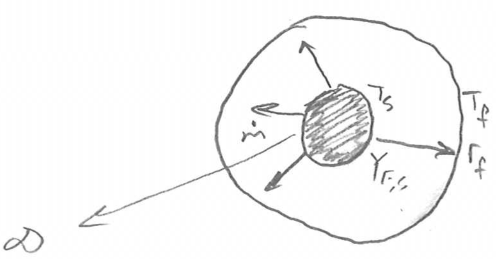
$$D^2 = D_0^2 - Kt$$

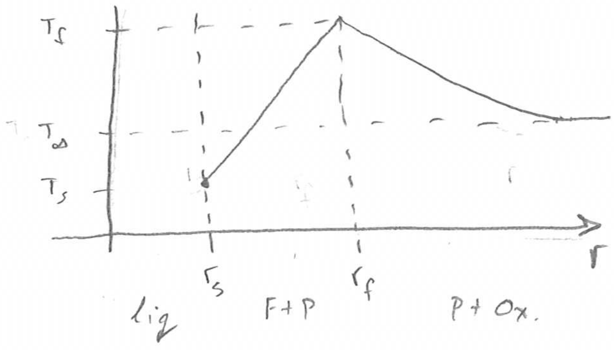
$$* \quad \boxed{t_d = D_0^2 / K}$$

- $C_p(\bar{T})$
- $\lambda = 0.4\,\lambda_F(\bar{T}) + 0.6\,\lambda_\infty(\bar{T})$
- $\bar{T} = \frac{1}{2}(T_\infty + T_s)$

Burning Droplets.

Same process, but Different B.C.





5 parameters

- $F + \nu\, Ox \rightarrow (1+\nu) Pr$
- Rxn at Flame Sheet
-

$$\dot{m}_F, \; Y_{F,s}, \; T_s, \; T_f, \; r_f$$

5 Equations.

- (Mass Conservation: $\dot{m}_F = \text{const}$)

(1) • Fuel Eqn in inner Region: $\dot{m}_F = -4\pi r^2 \dfrac{\rho \mathcal{D}}{1-Y_F} \dfrac{dY_F}{dr}$

$\longrightarrow Y_{F,s} = f(\dot{m}_F, r_f)$

(2) • Oxidizer in Outer Region: $\dot{m}_F = 4\pi r^2 \dfrac{\rho \mathcal{D}}{\nu + Y_{ox}} \dfrac{dY_{ox}}{dr}$

$\longrightarrow f(\dot{m}_F, r_f) = (\nu+1)/\nu$

• (Energy Eqn: Same as before: $\dfrac{d}{dr}\left(r^2 \dfrac{dT}{dr}\right) = \dfrac{\dot{m} c_p}{4\pi\lambda} \dfrac{dT}{dr}$

$\longrightarrow$ T(r) in inner, outer)

(3) • E Balance at Surf:

$$4\pi r^2 \lambda \left.\frac{dT}{dr}\right|_s = \dot{m}_F (h_{vap} + q_{liq})$$

$$\longrightarrow f(\dot{m}_F, T_f, T_s, r_f) = 0$$

(4) • E-bal at flame:

$$\dot{m}_F \Delta h_c = 4\pi r_f^2 \lambda \left(\left.\frac{dT}{dr}\right|_{r_f^-} - \left.\frac{dT}{dr}\right|_{r_f^+} \right)$$

$$\longrightarrow f(\dot{m}_F, T_f, T_s, r_f) = 0$$

(5) • Phase equilibrium:

$$Y_{F,s} = f(T_s)$$

See p. 390 for Solution procedure

1. Rearrange above
2. Guess T_s
3. Compute $\dot{m}_F, T_f, r_f, Y_{F,s} \longrightarrow$ implies T_s
4. Repeat to converge.