Energy Balance.

$$\frac{d(mu)_{sys}}{dt} = \dot{W} + \dot{Q} + \sum_i \dot{m}_i u_i$$

$$= \sum_i \dot{m}_i P v_i + \sum_j \dot{Q}_j + \sum_i \dot{m}_i u_i + \dot{W}_\phi$$

$\dot{W}_\phi \longrightarrow$ "other work" not including injection work

(1) $$\frac{d(mu)_{sys}}{dt} = \sum \dot{m}_i h_i + \dot{W}_\phi + \sum_j \dot{Q}_j$$

Entropy Balance.

$$\frac{d(ms)_{sys}}{dt} = \sum_i \dot{m}_i s_i + \underbrace{\sum_j \dot{S}_{j,HT}}_{\sum_j \dot{Q}_j/T_j} + \dot{S}_{irrev}$$

(2) $$\frac{d(ms)_{sys}}{dt} = \sum_i \dot{m}_i s_i + \sum_j \dot{Q}_j/T_j + \dot{S}_{irrev}$$

- The system exchanges heat with surrounding streams j at the surrounding stream temperature. In this way, all irreversibilities are w/i the system, and

$$\Delta \dot{S}_{sys} + \Delta \dot{S}_{surr} = \dot{S}_{irrev}$$

$$\dot{S}_{irr} = \frac{\dot{W}_L}{T_0} \quad \text{(show elsewhere)}$$

(3) (2) $\cdot T_0 \longrightarrow$ $$\frac{d(mT_0 s)_{sys}}{dt} = \sum_i \dot{m}_i T_0 s_i + \sum_j \dot{Q}_j \frac{T_0}{T_j} + \dot{W}_L$$

(1)−(3) : $$\frac{d(m(u-T_0 s))_{sys}}{dt} = \sum \dot{m}_i (h_i - T_0 s_i) + \sum_j \dot{Q}_j \left(1 - \frac{T_0}{T_j}\right) + \dot{W}_\phi - \dot{W}_L$$

$$\dot{W}_\phi = \left[\frac{d(m(u-T_0 s))_{sys}}{dt} - \sum_i \dot{m}_i (h_i - T_0 s_i) - \sum_j \dot{Q}_j \left(1 - \frac{T_0}{T_j}\right)\right] + \dot{W}_L$$

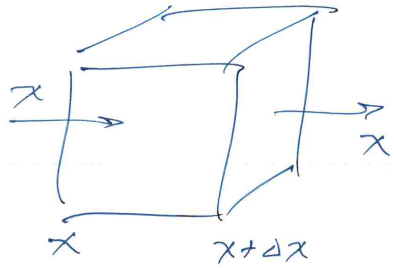
or $\dot{W}_\phi - \dot{W}_L = [\quad]$

if $\dot{W}_L = 0$ $\quad \dot{W}_\phi$ is reversible

$\dot{W}_{\phi,rev} = [\quad]$

Species i $\quad$ Acc = in - out + gen.

Y_i (2)



Y_i

Acc : $\dot{m}_i$ $\quad \frac{dm_i}{dt}$, $\frac{d\rho Y_i \cdot V}{dt}$

in - out : $(A \cdot v \cdot \rho \cdot Y_i)_x - (A v \rho Y_i)_{x+\Delta x}$

$m^2 \cdot \frac{m}{s} \cdot \frac{kg}{m^3} \cdot \frac{kg_i}{kg}$

$A = \Delta y \Delta z$ $\qquad\qquad \vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$

$$\frac{d(\rho Y_i \Delta x \Delta y \Delta z)}{dt} = (\Delta y \Delta z\, u \rho Y_i)_x - (\Delta y \Delta z\, u\rho Y_i)_{x+\Delta x}$$
$$+ (\Delta x \Delta z\, v \rho Y_i)_y - (\Delta x \Delta z\, v\rho Y_i)_{y+\Delta y}$$
$$+ (\Delta x \Delta y\, w \rho Y_i)_z - (\Delta x \Delta y\, w\rho Y_i)_{z+\Delta z}.$$
} advection.

$$+ (J_{i,x} \cdot \Delta y \Delta z)_x - (J_{i,x} \cdot \Delta y \Delta z)_{x+\Delta x}.$$
$$+ (\quad) - (\quad)$$
$$+ (\quad) - (\quad)$$
} Diff.

$$+ \dot{m}_i''' \cdot V$$
} gen.

$$\frac{d\rho Y_i}{dt} = \frac{(u\rho Y_i)_x - (u\rho Y_i)_{x+\Delta x}}{\Delta x} + \frac{(v\rho Y_i)_y - (v\rho Y_i)_{y+\Delta y}}{\Delta y} + \frac{(\quad) - (\quad)}{\Delta z}$$
$$+ \frac{J - J}{\Delta x} + \frac{J - J}{\Delta y} + \frac{J - J}{\Delta z} + \dot{m}_i''' \ldots$$

$$\frac{\partial \rho Y_i}{\partial t} = -\frac{\partial \rho Y_i u}{\partial x} - \frac{\partial \rho Y_i v}{\partial y} - \frac{\partial \rho Y_i w}{\partial z}$$

$$-\frac{\partial J_x}{\partial x} - \frac{\partial J_y}{\partial y} - \frac{\partial J_z}{\partial z} + \dot{m}_i'''$$

$$* \quad \frac{\partial \rho Y_i}{\partial t} + \vec{\nabla}\cdot(\rho Y_i \vec{v}) + \vec{\nabla}\cdot \vec{J}_{Y_i} = \dot{m}_i'''$$

$$\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

$$\vec{v} = u\vec{i} + v\vec{j} + w\vec{k}$$

$$\vec{\nabla}\cdot\vec{v}$$

in 1-D.

$$\frac{\partial \rho Y_i}{\partial t} + \frac{\partial}{\partial x}(\rho Y_i u) + \frac{\partial}{\partial x} J_x = \dot{m}_i'''$$

Acc + out-in + out-in = gen

$$\frac{\partial \rho Y_i}{\partial t} + \frac{\partial}{\partial x}(\rho Y_i u_i) = \dot{m}_i'''$$

$$u_i = u + u_i^D$$

$$u_i^D = u_i - u$$

$$u = \sum Y_i u_i = \frac{\sum m_i u_i}{\sum m_i}$$

$$J_i = \rho Y_i u_i^D = -\rho D_i \frac{\partial Y_i}{\partial x}$$

$\circledast$

Energy balance.

$$dU = dQ + dW$$

$$d(mu)_{sys} = dQ + dW + \sum u_i \, dm_i$$

$$dW = \sum p v_i \, dm_i + dW_{other}$$

$$d(mu)_{sys} = dQ + dW_o + \sum u_i \, dm_i + \sum p v_i \, dm_i$$

$$\circledast \quad d(mu)_{sys} = dQ + dW_o + \sum h_i \, dm_i$$

Entropy balance.

$$\circledast \quad d(ms)_{sys} = \frac{dQ}{T_{HT}} + \sum s_i \, dm_i + \underbrace{dS_{irrev.}}_{\frac{d(LW)}{T_o}}$$

$$\text{Energy:} \quad dW_o = d(mu)_{sys} - dQ - \sum h_i \, dm_i$$

$$T_o \cdot \text{Entropy:} \quad dW_L = T_o \, d(ms)_{sys} - \frac{T_o}{T_{HT}} dQ - \sum T_o s_i \, dm_i$$

$$dW_o - dW_L = \left[d\left(m(u - T_o s)_{sys}\right) - \left(1 - \frac{T_o}{T_{HT}}\right) dQ - \sum_i (h_i - T_o s_i) \, dm_i \right]$$

$$\circledast \quad dW_{Rev} = \left[d\left(m(u - T_o s)_{sys}\right) - \left(1 - \frac{T_o}{T_{HT}}\right) dQ - \sum_i (h_i - T_o s_i) \, dm_i \right]$$

or $dW_o = dW_{Rev} + dW_L$

Work = Reversible Work + Lost Work

$\ast$

Why is $LW = T_0 \Delta S$?

Suppose we have a hot sys T & a cold dead state.

if we transfer heat at T_h :

Then we have energy available at T_h.

Call that energy $U = Q_{\text{transferred at } T_h} = T_h \Delta S$

but then, we can run a ~~heat~~ rev. heat engine ~~us~~ ~~the~~ using that energy.

$$W = U\left(1 - \frac{T_c}{T_h}\right) = Q\left(1 - \frac{T_c}{T_h}\right)$$

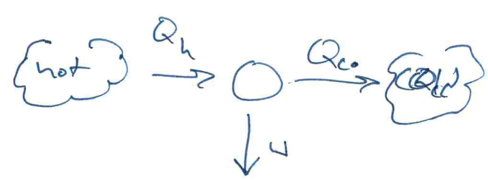
$$= T_h \Delta S \left(1 - \frac{T_c}{T_h}\right)$$

$$= T_h \Delta S - T_c \Delta S$$

$$W = Q - T_c \Delta S.$$

in transferring $Q = U$ units of energy at T_h, we can still do W units of useful work, with $T_c \Delta S$ as the lost work.

Carnot: E bal : $Q_h = W + Q_c$



$$\rightarrow W = Q_h - Q_c$$

$$= Q_h\left(1 - \frac{Q_c}{Q_h}\right)$$

$$= Q_h\left(1 - \frac{T_c \cancel{\Delta S}}{T_h \cancel{\Delta S}}\right)$$

$$* \; W = Q_h\left(1 - \frac{T_c}{T_h}\right)$$

Reversible proc. Don't generate entropy $\rightarrow$

$$\Delta S_h = T_h Q_h = \Delta S_c = T_c Q_c$$

Example hot, high P steam → Cold, low P liq.

(1) $T = 500 F$
$P = 200$ psia
$H = 1268.9$ Btu/lbm
$S = 1.624$ Btu/lbm·°R

(2) $T = 77 F$
$P = 14.696$ psi
$H = 45.02$ Btu/lbm
$S = 0.0876$ Btu/lbm·°R

$\Delta H = W + Q = 1268.9 - 45.02 = 1223.88$ Btu/lbm.

Transfer minimum Q ; $Q = TdS$ → min T

$Q = T_2 \Delta S = (459.67 + 77)\,°R \cdot 1.5364$ Btu/lbm·°R

→ $\boxed{\begin{array}{l} Q = 824.54 \text{ Btu/lbm} \\ W = 399.34 \text{ Btu/lbm} \end{array}}$

How? (A) isentropic steam turbine ; $\Delta S = 0$, $P_1 \to P_2$
(B) Reversible heat engine.

(A) (1) → (1b)

(1)	(1b)	
T=500	T=?	T= 212.05
P=200 psi	P=14.696	
H = 1268.9	H = ~~[illegible]~~ ?	H= 1061.36
S= 1.624	S = 1.6240	

Cantera. S= 3520 J/kg·K ; H = -1.5971 E7,

ST: S=0, H=0

$W = \Delta H = 207.54$ Btu/lbm. , $Q = 0$

(B) (1b) → (2)

(1b)	(2)
T = 212.05 °F	T= 77 °F
P = 14.696	P= 14.696 psi
H = 1061.36	H = 45.02
S= 1.624	S = 0.0876

Reversible H. Engine

$\Delta H = W + Q$

maximize W, minimize Q. $\Delta Q = T\Delta S$.

→ Transfer heat at min T

→ Transfer heat at T_2

$$Q = T_2 \Delta S$$
$$= (459.67 + 77)(1.624 - 0.0876)$$
$$= \cancel{866.85}\ 824.54 \text{ Btu/lb.}$$

$\Delta H = 1061.36 - 45.02 = 1016.34$ Btu/lb.

$W = \Delta H - Q = \cancel{[illegible]}\ 191.8$ Btu/lb

$W_A + W_B = 399$ Btu/lb.

$\Delta H_{1b,2} = 1061.36 - 45.02 = 1016.34$

$\Delta S_{1b,2} = 1.624 - 0.0876 = 1.5364$

$\Delta H = W + \underbrace{Q}_{\text{Lost work.}}$

$W = \Delta H - Q$

$= 1016.34 - \underbrace{T_2 \Delta S}_{(77+459.67)(\Delta S)}$

$= 191.8$

~~Lagrangian form.~~

Eulerian (above)

- Const box.

mass: $\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0$

mom: $\frac{\partial \rho\vec{v}}{\partial t} + \nabla\cdot(\rho\vec{v}\vec{v}) = -\nabla P - \nabla\cdot\underline{\underline{\tau}} + \nabla\cdot\rho\vec{g}$

spec: $\frac{\partial \rho Y_k}{\partial t} + \nabla\cdot(\rho Y_k\vec{v}) = -\nabla\cdot\vec{j} + \dot{m}_k'''$

Lag. 1-D.

mass; ~~m in a cell~~ $\rho\Delta x = \text{const}$.

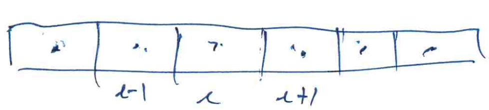
~ (oops)

mom: $\frac{\partial v}{\partial t}$

Numerical Solution.

1-D ~~mass~~ ~~$\partial\rho$~~

Species: $\frac{\partial \rho Y_k}{\partial t} = -\frac{\partial(\rho Y_k u)}{\partial x} - \frac{\partial j_x}{\partial x} + \dot{m}_k'''$



Finite Diff

Finite Volume.

FD; $\left(\frac{\partial\phi}{\partial x}\right)_i \approx \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x}$

$$\frac{\partial \rho Y_{k,i}}{\partial t} = -\left[\frac{(\rho Y_k u)_{i+1} - (\rho Y_k u)_{i-1}}{2\Delta x}\right] - \left[\quad\right] + \dot{m}_{k,i}'''$$