Chemical Engineering 374

Fluid Mechanics
Fall 2013

Bernoulli Equation

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"Generation" Accumulation Out In

$$\frac{dQ}{dt} + \frac{dW_s}{dt} = \frac{d}{dt}\left[\underbrace{\rho(u + \frac{1}{2}v^2 + gz)}_{e}V\right] + \left[\rho v A(\underbrace{u + \frac{P}{\rho} + \frac{1}{2}v^2 + gz}_{e_{mech}})\right]_{out} - []_{in}$$

Can rearrange to familiar **(Accumulation) = (In) – (Out) + ("Generation")**

<u>Simplify</u>

- Steady State
- Ws = 0
- Q = 0
- No friction (viscous effects)
    - This and no Q give const. u
- Incompressible → constant density

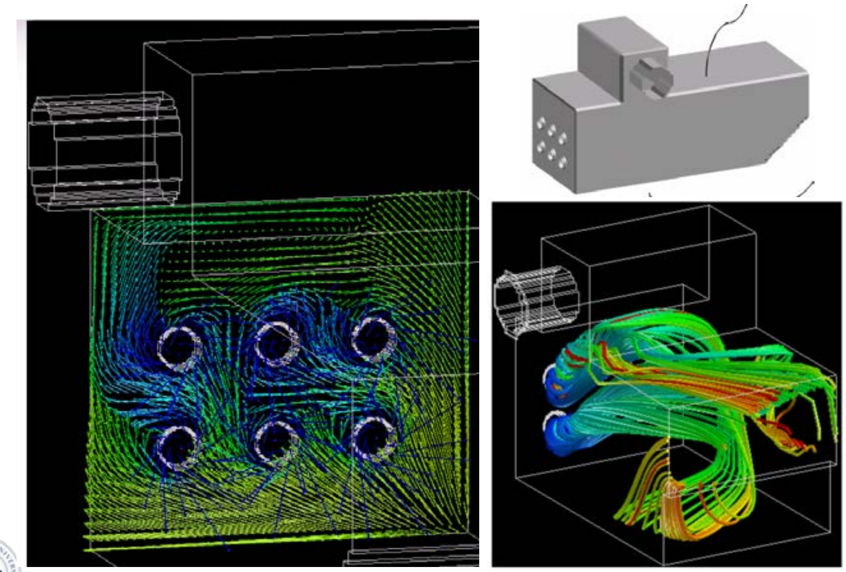
$$\left(\frac{P}{\rho} + \frac{1}{2}v^2 + gz\right)_{in} = \left(\frac{P}{\rho} + \frac{1}{2}v^2 + gz\right)_{out}$$

Or

$$\Delta\left(\frac{P}{\rho} + \frac{1}{2}v^2 + gz\right) = 0$$

e_{mech} is conserved

Streamlines



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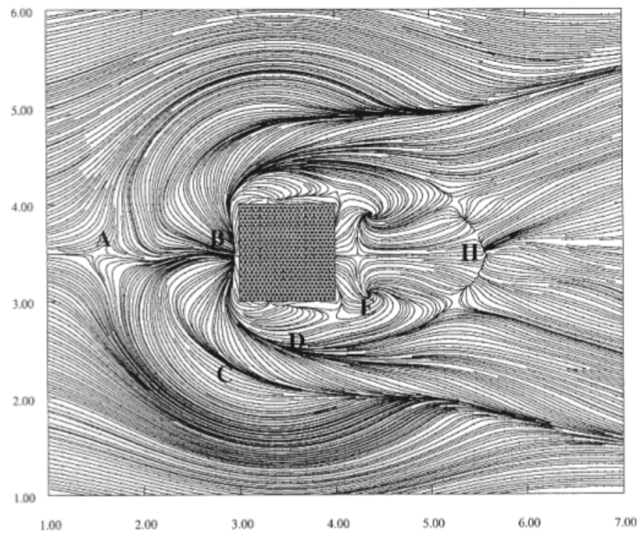

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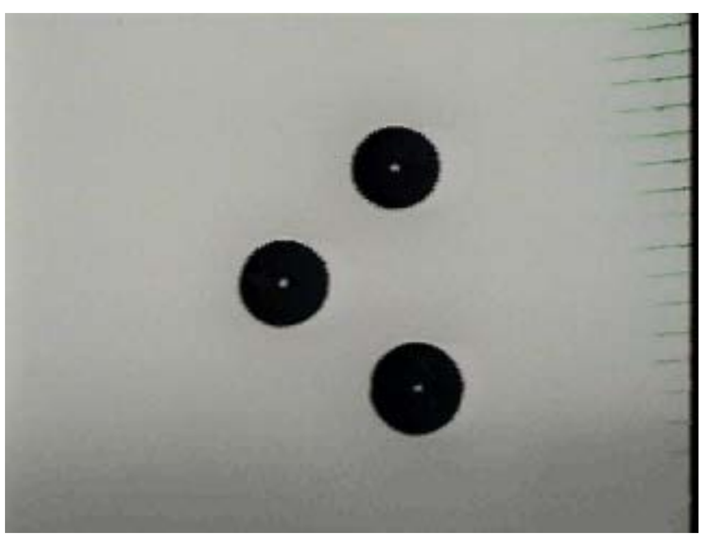

- Consider streamlines, then mechanical energy on a streamline is constant.
- Can derive the Bernoulli equation by making the same set of assumptions and "dot" the momentum equation (force balance equation, not covered yet) with displacement along a streamline.
- Cengel and Boles give a simpler derivation in terms of Newton's Second Law (force balance), again along a streamline.
- Other forms of Bernoulli's equation exist
  - Unsteady
  - Compressible
  - As usual, back up in the derivation when making assumptions.

Bernoulli Equation and Pressure

$$\left(\frac{P}{\rho} + \frac{1}{2}v^2 + gz\right) = C$$

Units

$$\frac{P}{\rho} (=) \frac{N}{m^2 \cdot kg/m^3} (=) \frac{kg \cdot m}{s^2 \cdot m^2 \cdot kg/m^3} (=) \frac{m^2}{s^2}$$
$$(=) \frac{J}{kg} (=) \frac{kg \cdot m^2}{kg \cdot s^2}$$

B.E. units are energy per unit mass
But since the mechanical energy is constant, can multiply through by density to give units of pressure.

$$\left(P + \frac{1}{2}\rho v^2 + \rho gz\right) = C$$

Static | Dynamic | Hydrostatic

Total pressure

Application

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- You are an airplane.
- Measure your velocity.
- How?
- Apply Bernoulli Equation $\left(\frac{P}{\rho}+\frac{1}{2}v^2+gz\right)_1=\left(\frac{P}{\rho}+\frac{1}{2}v^2+gz\right)_2$
- You have a bunch of variables
  - One is unknown.
  - The rest are either: *known*, *measured*, *controlled*
  - You have a constraint, what is it?

Pitot Tube

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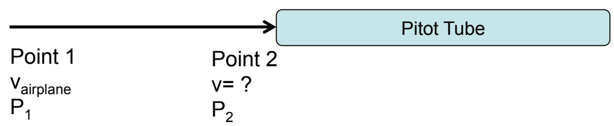
Pitot Tube



- Note the correlation between points and the device.
- Note the streamline.
- Note the control over v_2
- What is the principle: how does it work?

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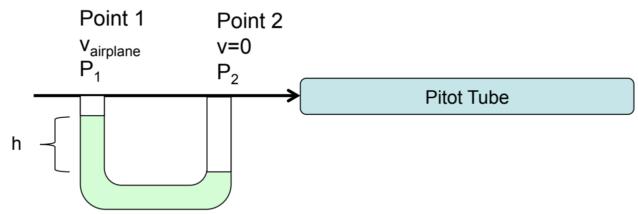
Velocity Measurement

- Velocity measurement
- Total pressure is constant along a streamline
- Measure pressure at two points on the same streamline
  - Where the velocity is desired
  - At a point where the velocity has stagnated
- $P_{stagnation} = P_{static} + P_{dynamic}$
- Stagnation pressure is the pressure to bring the fluid to zero velocity without friction.

$$\left(\frac{P}{\rho} + \frac{1}{2}v^2 + gz\right)_1 = \left(\frac{P}{\rho} + \frac{1}{2}v^2 + gz\right)_2$$

$$\left(P + \frac{1}{2}\rho v^2\right)_1 = \left(P + \frac{1}{2}\rho v^2\right)_2 \qquad v = \sqrt{\frac{2}{\rho}(P_2 - P_1)}$$

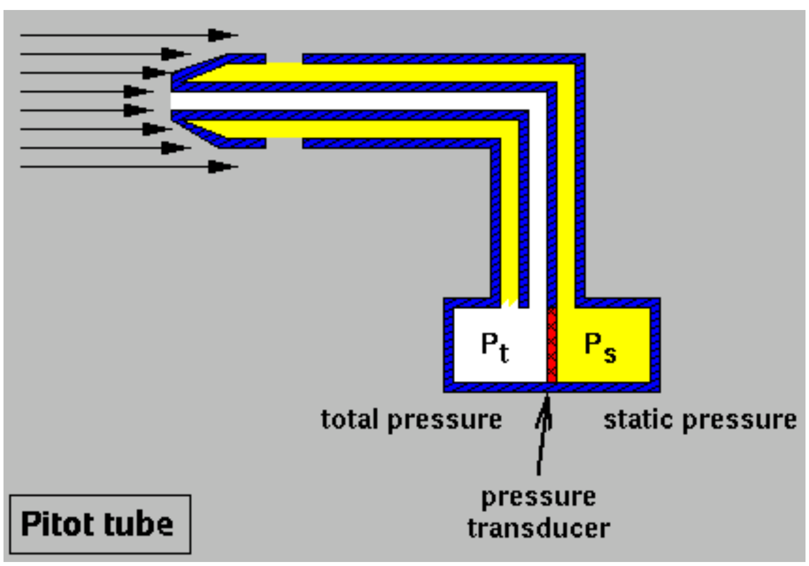
How to measure P_2-P_1



$$P_2 - P_1 = \rho g h$$

- Use a manometer,
- Or a pressure transducer, etc.
- Note, the real device is not laid out like this, but is analysed like this.

Pitot Tube



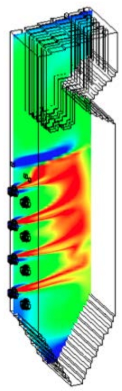
Velocity Measurement

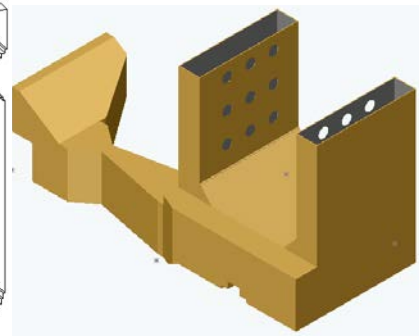
- Problem solving with the Bernoulli equation amounts to:
  - Splitting configuration into points, evaluating P,v,h at one point and two of P,v,h at the other, and solving for the unknown with B.E.
  - Countless examples, all boil down to this.
  - Often involve multiple applications → two B.E. in two unknowns.

- Real flows are not ideal, and have friction losses.
- Friction results in a variation in internal energy (u).
- Rather than include Δu, include a friction loss term *F*
- For constant height and velocity, friction causes pressure drop.
  - Bigger fans, pumps, turbines needed for the same flow!
  - Minimize the pressure drop (friction).

$$\Delta\left(\frac{P}{\rho}+\frac{1}{2}v^2+gz\right)=0 \quad \Longrightarrow \quad \Delta\left(\frac{P}{\rho}+\frac{1}{2}v^2+gz\right)=-F$$

Powerplant windbox

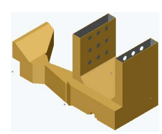


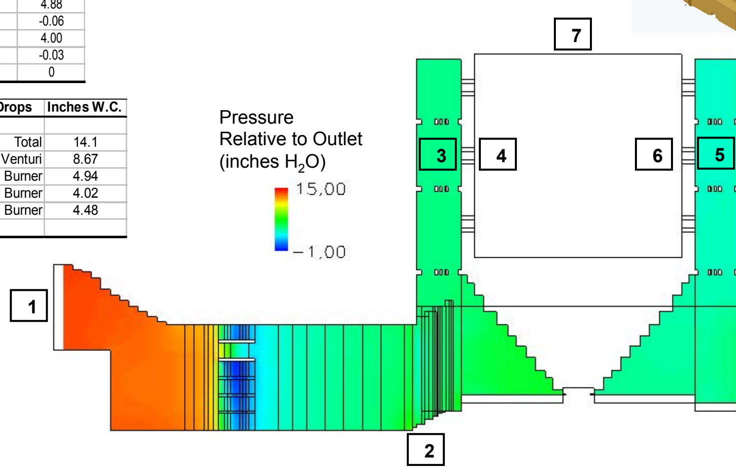


Windbox Pressure Drop

Location *	Inches W.C
1	14.1
2	5.44
3	4.88
4	-0.06
5	4.00
6	-0.03
7	0

Pressure Drops	Inches W.C.
Total	14.1
Venturi	8.67
RW Average Burner	4.94
FW Average Burner	4.02
Average Burner	4.48



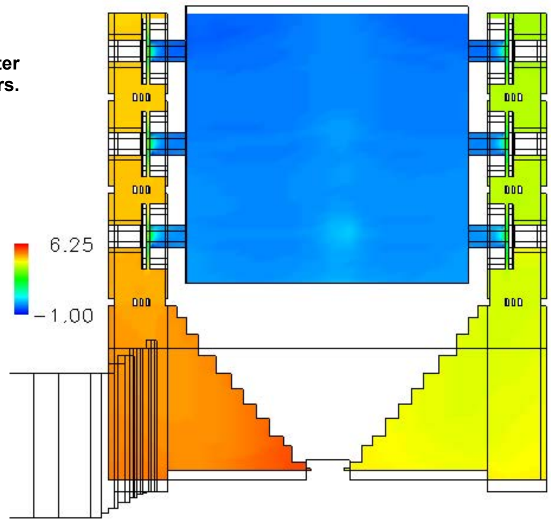


Windbox Pressure Profile

Side view through the center column of burners.

Pressure Relative to

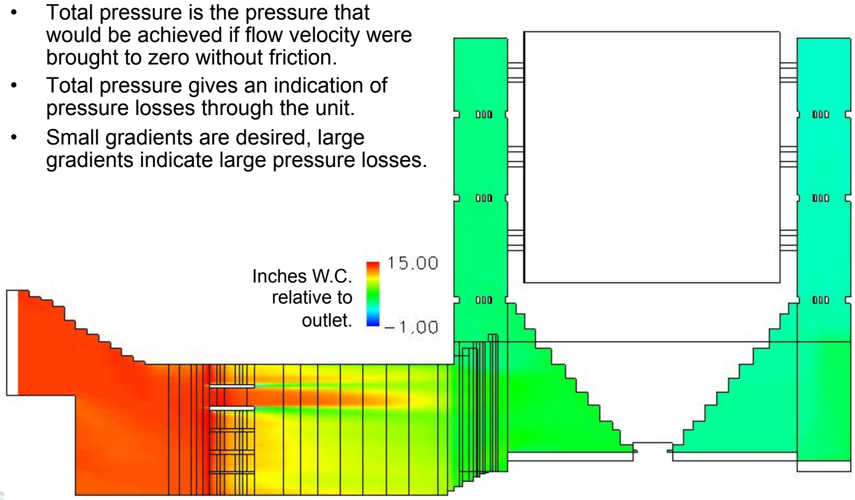
Outlet (inches H_2O)



Total Pressure

- Total pressure is the pressure that would be achieved if flow velocity were brought to zero without friction.
- Total pressure gives an indication of pressure losses through the unit.
- Small gradients are desired, large gradients indicate large pressure losses.

Inches W.C. relative to outlet.



Total Pressure

View through center burners.

Inches W.C. relative to outlet.