Chemical Engineering 374

Fluid Mechanics
Fall 2011

Integral Energy Balance

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In this house, we obey the laws of thermodynamics!

– *Homer Simpson*

Integral Energy Balance

- Recall, we are writing balance equations.
  - Fluid Statics (no flow)
  - Mass Balance (last time)
  - Energy Balance (today)
  - Momentum Balance (later)

Reynolds Transport Theorem

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b dV + \int_{CS} \rho b \vec{v} \cdot \vec{n} dA$$

System of fixed Mass (closed system)

Control Volume: Some (usually fixed) region of space

Energy of the system: Internal, Kinetic, Potential

$$E = U + \frac{1}{2}mv^2 + mgz$$

$$e = u + \frac{1}{2}v^2 + gz$$

Conservation law for our system? 1st Law of thermodynamics

$$\frac{dE}{dt} = \frac{dQ}{dt} + \frac{dW}{dt}$$

RTT → B_{sys} = E, b = e

$$\frac{dQ}{dt} + \frac{dW}{dt} = \frac{d}{dt}\int_{CV} \rho(u + \frac{1}{2}v^2 + gz)dV + \int_{CS} \rho(u + \frac{1}{2}v^2 + gz)\vec{v} \cdot \vec{n} dA$$

$\rho e dV$ Volumetric energy

$\rho e \vec{v} \cdot \vec{n} dA$ Energy flux

Consider Uniform Properties within a control volum, and Uniform Velocities

$$\frac{dQ}{dt} + \frac{dW}{dt} = \frac{d}{dt}\left[\rho(u + \frac{1}{2}v^2 + gz)V\right] + \left[\underbrace{\rho v A}_{\dot{m}}(u + \frac{1}{2}v^2 + gz)\right]_{out} - \left[\rho v A(u + \frac{1}{2}v^2 + gz)\right]_{in}$$

Pressure is buried in the work term

$$\frac{dW}{dt} : \quad dW = \vec{F} \cdot d\vec{x}$$

Forces at the Surface

- **W_p** – Pressure forces (stress)
- W_v – Viscous stresses (usually ignore as small)

Forces internal to the system

- **W_s** – Shaft work (pump, turbine)
- W_o – Other (electric, magnetic, surface tension)

W_s is left as is → either specified directly, or computed

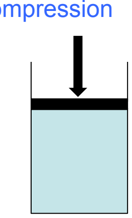
Positive when work is done on the system

Negative when system does work on surroundings

W_p is pressure work, or work to deform the boundary of the SYSTEM

$$dW = Fdx = PAdx$$

Consider piston compression



$$\frac{dW}{dt} = PA\frac{dx}{dt} = PAv$$

$$\frac{dW}{dt} = -\int_{CS} P\vec{v} \cdot \vec{n} dA = -\int_{CS} \frac{P}{\rho}\rho\vec{v} \cdot \vec{n} dA$$

General control volume

- P/ρ (=) energy per mass
- Note the negative sign
- This work is the rate of energy flux across the system surface
- associated with pressure work (deformation).
- This is the rate of energy needed to move the fluid (to move the system)

$$\frac{dQ}{dt} + \frac{dW_s}{dt} - \int_{CS} \frac{P}{\rho}\rho\vec{v}\cdot\vec{n}dA = \frac{d}{dt}\int_{CV}\rho(u+\frac{1}{2}v^2+gz)dV + \int_{CS}\rho(u+\frac{1}{2}v^2+gz)\vec{v}\cdot\vec{n}dA$$

- •Move term to RHS
- •Assume uniform properties

$$\frac{dQ}{dt} + \frac{dW_s}{dt} = \frac{d}{dt}\left[\rho(u+\frac{1}{2}v^2+gz)V\right] + \left[\rho vA(u+\frac{P}{\rho}+\frac{1}{2}v^2+gz)\right]_{out} - []_{in}$$

- •Multiple streams need multiple terms
- •u+P/ρ = h = u+P*v*

$$\frac{dQ}{dt} + \frac{dW_s}{dt} = \frac{d}{dt}\left[\rho(u+\frac{1}{2}v^2+gz)V\right] + \left[\rho vA(u+\frac{P}{\rho}+\frac{1}{2}v^2+gz)\right]_{out} - []_{in}$$

Simplify

- •Steady State
- •Q=0 (no heat transfer)
- •Constant mass flow
- •Constant internal energy (no friction, ΔT, Q)

$$\frac{dW_s}{dt} = \left[\rho vA(\frac{P}{\rho}+\frac{1}{2}v^2+gz)\right]_{out} - \left[\rho vA(\frac{P}{\rho}+\frac{1}{2}v^2+gz)\right]_{in}$$

$$= \dot{m}\Delta e_{mech} = \Delta\dot{E}_{mech}$$

e_{mech}

- •Shaft work converted to mechanical energy.
- •Mechanical energy is the energy that can be directly converted to mechanical work.
- •Ideal, no losses (friction/heat)
- •Real systems have losses
- •Convenient to consider the ideal case with some efficiency: known or compute.

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$$\eta = \frac{E_{mech,\,real}}{E_{mech,\,ideal}}$$

$$\eta_{pump} = \frac{\Delta E_{mech}}{W_{shaft}}$$

$$\eta_{turbine} = \frac{W_{shaft}}{\Delta E_{mech}}$$

•Efficiency is positive, so use absolute values if needed.
•Pump/motor, turbine/motor → product of efficiencies

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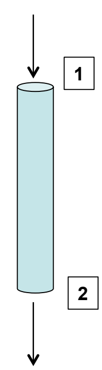
Example 1

Frictionless, Steady, Vertical Pipe Flow

$$\frac{dQ}{dt} + \frac{dW_s}{dt} = \frac{d}{dt}\left[\rho(u + \frac{1}{2}v^2 + gz)V\right] + \left[\rho v A(u + \frac{P}{\rho} + \frac{1}{2}v^2 + gz)\right]_{out} - []_{in}$$

0 0 0



Δu?
Δv?
Δm?

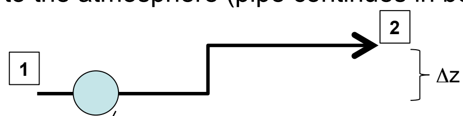
$$\left(\frac{P}{\rho} + gz\right)_{out} = \left(\frac{P}{\rho} + gz\right)_{in}$$

$$(P_2 - P_1) = -\rho g(z_2 - z_1) = \rho g h$$

Our old friend!

Example 2

- Pump liquid through a steady, frictionless nozzle
  - Nozzle, so A_1 not equal to A_2
- Not open to the atmosphere (pipe continues in both directions



$$\frac{dQ}{dt} + \frac{dW_s}{dt} = \frac{d}{dt}\left[\rho(u + \frac{1}{2}v^2 + gz)V\right] + \left[\rho v A(u + \frac{P}{\rho} + \frac{1}{2}v^2 + gz)\right]_{out} - []_{in}$$

0

0

Δu?
Δv?
Δm?

$$\frac{\dot{W}_p}{\dot{m}} = \frac{P_2 - P_1}{\rho} + \frac{v_2^2 - v_1^2}{2} + g(z_2 - z_1)$$

What if ends open to the atmosphere?
What if the inlet and outlet pipes are the same size?