ChE 374–Lecture 4–Pressure: Buoyancy, Measurements, Surfaces

- Buoyancy
  - Fluid statics → just a FORCE BALANCE!
  - Buoyant force is the Net upward force
    * F_b is the weight of the displaced fluid.
    * $F_b = \rho_a g A h_1 + \rho_w g A h_2$ (for a partially submerged object in air and water with h_1 the part in the air and h_2 the part in the water.)
- Pressure Measurement
  - Barometer
    * Measures atmospheric pressure.
    * $P_{atm} - P_0 = \rho g h$.
    * Account for temperature, partial pressure.
    * P_{atm}=760 mmHg at 0 ^{o}C, at $g = 9.807$ m/s^2.
    * $< 1\%$ variation at 20 ^{o}C.
  - Bourdon Tube
    * Tube is curved → outside surface area > inside surface area.
    * $F = PA$ → force is higher on outside → tube unravels.
    * Rugged, cheap, reliable.
    * Most common industrial pressure Gage, (measures gage pressure).
  - Others:
    * Strain gage → stretch leads to variation in electrical output.
    * Pressure transducer → current via voltage, capacitors, resistors.
    * Piezoelectric → variation in voltage via pressure.
  - Manometer
    * Fluid columns → measure a ΔP.
      · Like a U with a working fluid at the bottom separating two sides with pressure to be measured.
    * Pressure at the bottom is the same from one side or the other so write the statics expression on the bottom for both sides and equate and solve for the pressure difference.
    * Or work progressively from one side to another.
    * Fluid choices considerations.
    * Recall: points at the same height in the same fluid are at the same P.
      · They have the same weight above them, otherwise the fluid would move.
- Surface forces

  $\vec{F}_s = -\int_{surf} P\vec{n}dA$.

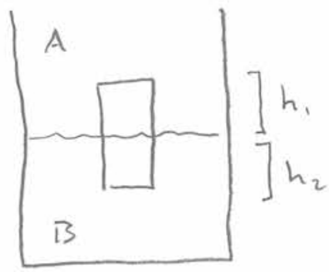
    * If P is constant then $|\vec{F}_s| = PA$.
    * If P is not constant, the use the Barometric equation to get P(h) and integrate.
  - Example: force on a dam → Force is just P at middle * Area.
  - In general, evaluate at the centroid.
    * h_c is the area-weighted average height.
    * In general: $\bar{f(x)} = \frac{1}{X}\int_0 X f(x)dx$.
    * $h_c = \frac{1}{A}\int_0^H h dA$.
      · h_c is the h where half the area is above and half is below.

Class 4

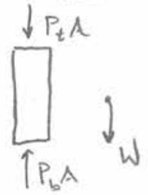
- Buoyancy
- Pressure measurement
- Pressure Forces on Surfaces.

Buoyancy

- Net upward Force by a fluid on a Submerged object.
- Fluid statics → Force Balance.
- Consider:



$\Sigma F = 0$



$$\underbrace{P_b A - P_t A}_{F_b} - W = 0$$

$F_b = P_b A - P_t A \quad ; \quad P_b = P_t + \rho_A g h_1 + \rho_B g h_2$

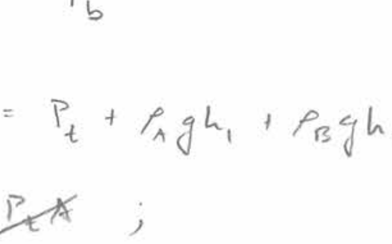
$P_b = \cancel{P_t A} + \rho_A g h_1 + \rho_B g h_2 - \cancel{P_t A} \quad ;$

* $F_b = \rho_A g h_1 A + \rho_B g h_2 A$ = weight of Displaced fluid

- If Fluid A is gas → $F_b \approx \rho_B g h_2 A$
- $F_b = W$ only if object is Floating

Pressure Measurement

Barometer – See $\boxed{\text{Slide}}$



$P_{atm} - P_o = \rho g h$

- P_o is vapor pressure @ Temp
- P_{atm} = 760 mm Hg at 0°C at $g = 9.807\ m/s^2$

  < 1% variation at 20°C
- P_{atm} = 406 in H_2O = 33.9 ft !

Bourdon Tube

- Curved tube
- $A_o > A_i \rightarrow F = PA \rightarrow F_o > F_i \rightarrow$ unravel
- Measures Gage pressure
- Rugged, Cheap, Reliable, Common.

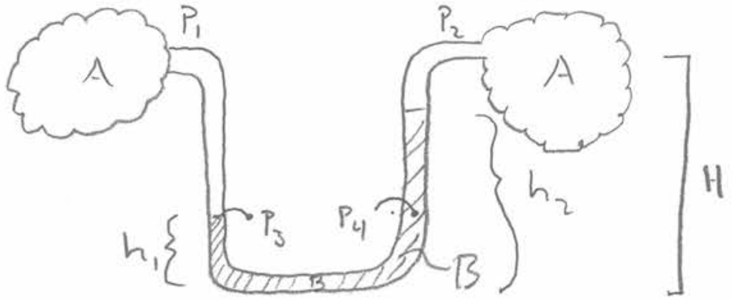
Others

Strain gage
Pressure Transducer
Piezoelectric

Manometer

Fluid Column $\rightarrow$ measures a ΔP

- Small to moderate ΔP
- Common in Labs



- Start at P_1, use $\Delta P = \rho g h$ to get to P_2:

$$P_1 + \rho_A g(H - h_1) + \rho_B g h_1 - \rho_B g h_2 - \rho_A g(H - h_2) = P_2$$

- Rearrange

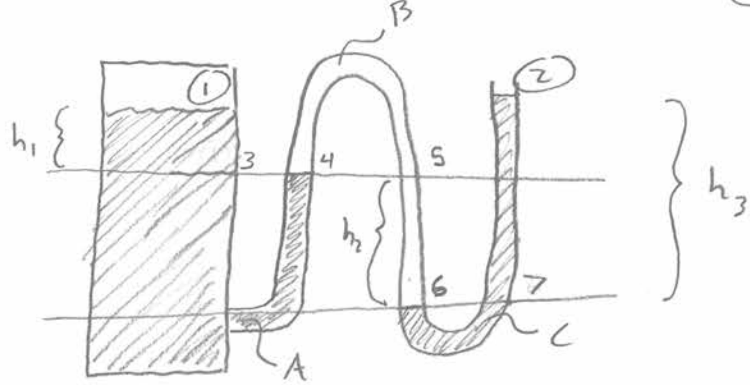
$$P_2 - P_1 = (\rho_B - \rho_A) g (h_2 - h_1)$$

$$* \quad \Delta P = \Delta\rho\, g\, \Delta h$$

- If A is gas, $\Delta P = \rho_B g (h_2 - h_1)$
- We can Skip Steps by noting $P_3 = P_4$:

$$P_1 + \rho_A g(H - h_1) - \rho_B g(h_2 - h_1) - \rho_A g(H - h_2) = P_2$$

Book Example 3-22



$$P_3 = P_4 \quad P_4 = P_5 \quad \Big\} \; P_3 = P_5$$

$$P_6 = P_7$$

$$* \quad P_2 = \underbrace{P_1 + \rho_A g h_1}_{1\to 3} + \underbrace{\rho_B g h_2}_{5\to 6} + \underbrace{\rho_C g h_3}_{7\to 2}$$

$3=5$ hop

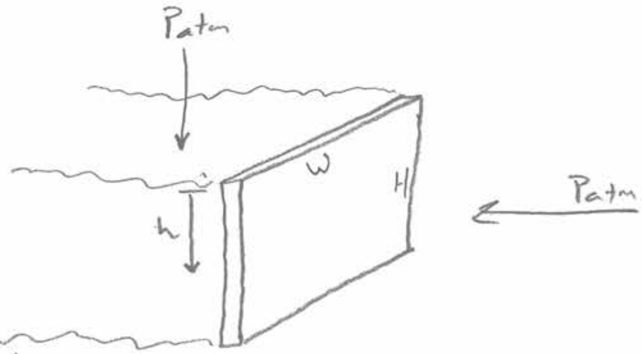
$6=7$ hop

- Take advantage of Points at Same height in Continuous fluid, at Same P

- Legs can be <u>inclined</u> → increased resolution.

- Choose fluid : Gas / Liq. ; Imiscible Liquids, Toxicity (Hg)

Pressure Forces on Surfaces

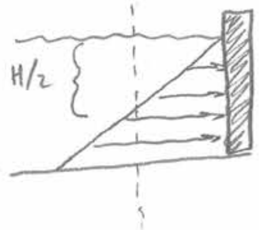


- $\vec{F}_s = -\int P\,\vec{n}\,dA$
- Example : Force on Dam ?
  - Pressure varies with height.
  - $F = PA \rightarrow dF = PdA = (P)W\,dh \;;\; P = \rho g h$
  - Atmosphere on Both Sides → Cancels

$$\int_0^F dF = \int_0^H (\rho g h)\,W\,dh$$

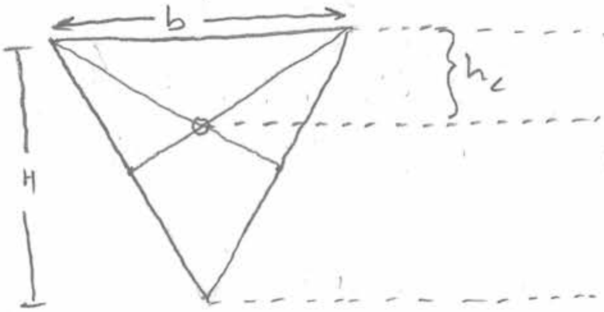
$$\boxed{F = \rho g W H^2/2}$$

- Note $F = \dfrac{\rho g W H^2}{2} = \left(\rho g \dfrac{H}{2}\right)(HW) = \left(P_{\text{at } h = \frac{H}{2}}\right)(A)$
- That is, The Force is $F = PA$ with $P = \rho g h$ evaluated at $h = \dfrac{H}{2}$ $\longrightarrow$ Centroid = Area-weighted avg height.



- $\dfrac{H}{2}$ Gives the avg Pressure.
- $h_c = \dfrac{1}{A}\int h\,dA = \dfrac{1}{\cancel{W}H}\int_0^H \cancel{W} h\,dh = \dfrac{H}{2}$

Example



High A Low P

Lower A, Higher P

- Centroid is The balance point
- mean P is at $\dfrac{H}{2}$, but more area in top half Than bottom $\longrightarrow$ Bias higher

- $F = \int P\,dA = \int \rho g h W(h)\,dh = \int \rho g h\left(b - \dfrac{hb}{H}\right)dh$

$$F = \left(\frac{\rho g b h^2}{2} - \frac{\rho g b h^3}{3H}\right)\Bigg|_0^H = \frac{\rho g b H^2}{2} - \frac{\rho g b H^3}{3H} = \frac{\rho g b H^2}{6}$$

- $F = \rho g h_c A = \rho g h_c \left(\dfrac{Hb}{2}\right) = \dfrac{\rho g b H^2}{6}$

$\longrightarrow$ $\boxed{h_c = \dfrac{H}{3}}$

Chemical Engineering 374

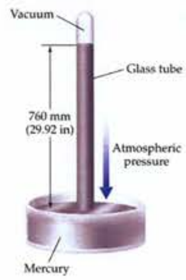
Fluid Mechanics
Fall 2011

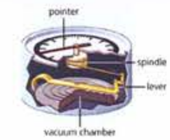
Pressure and Fluid Statics

1

Barometer

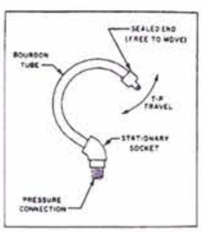
2





Bourdon Tube

3



Manometer

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