Chemical Engineering 374

Fluid Mechanics

Exam 3 Review

Exam Review, By Content/Lectures

- Classes 25-32 (plus review)
- Environmental
- Chapter 6.1-6.4 Momentum Balance
- Chapter 9.1-9.2, 9.4-9.6 Differential Balances
- Chapter 10.6 Boundary Layers
- Chapter 11.1-11.6 External Flows: Drag
- Chapter 14.1-14.5 Pumps and Turbines
- HW 19-27

Class 25—Integral Momentum Balance

- Force Balance

$$\sum \vec{F} = \frac{dm\vec{v}}{dt} + \sum(\dot{m}\vec{v})_{out} - \sum(\dot{m}\vec{v})_{in}$$

  - mdot is not a vector
  - Forces are external: body and surface (esp. P)
  - Decompose into scalar directions
    - One equation in one unknown (force, etc.)
    - Signs matter!
- Pick control volume intelligently
- Pressure surface forces should be gauge (unless account for full absolute pressure (including atm) on all surfaces.)
- For nonuniform flows, may have to integrate to get the full terms in the force balance, but the principle is the same.

Class 26—Differential Balances

- Mass Balance
- Momentum Balance
- Derived from the RTT
- Continuity Equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \qquad \nabla \cdot \vec{v} = 0$$

  - Used as a constraint for velocity
  - Book/handouts give expanded forms for Cartesian, Cylindrical, Spherical coordinates.
  - Incompressible (const ρ) means rate in = rate out.
  - If rate in = is not rate out then the density is changing.
- Momentum equation: just a differential force balance
  - (accum) = (in)-(out) + (body and surface forces)
  - Book/handouts give expanded forms in three coordinates
  - Again, use to find velocity field or pressure field.

Class 27—Navier-Stokes Equations

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\frac{1}{\rho}\nabla P + \nu \nabla^2 \vec{v} + \vec{g}$$

- Incompressible, Newtonian, vector
- The equations look scary, but we seriously simplify them
  - – Know how to drop out terms:
- Solution procedure:
  - – Is it steady? (usually, here), is it compressible (no, usually here const ρ), is it 1-D? (yes, usually here).
  - – 1D → one momentum equation (one component of the vector expression).
  - – 1D → maybe vary in y direction, but may have only u_x velocity (still 1-D)
    - Use only x-momentum, but varies in y-direction.
  - – Solve for a velocity component OR…
  - – Given a velocity field, can solve for the pressure field.
- Boundary conditions
  - – Velocity (mass flow) given at some point (inflow/outflow)
  - – Velocity zero at walls.
  - – Symmetry (like at centerlines) → velocity gradients are zero.
  - – Air/liquid interface → no shear → velocity gradient zero.

Class 28—Boundary Layers

- Mostly conceptual here.
  - – Euler equations (Navier-Stokes without viscosity) far from walls.
  - – Boundary layer equations near walls (Simplified Navier-Stokes equations).
- Boundary Layer results.
  - – Boundary layers have two length scales (length along the plate, and thickness).
  - – Boundary layers are very thin compared to length. (So Euler equations solved ignoring BL thickness)
  - – Thin, relatively parallel streamlines imply no pressure gradient normal to the plate. (Then the pressure gradient along the plate is that in the free stream (=0 for flat plate BL).
  - – Navier-stokes (or x-mom) told us there are two length scales.
  - – Continuity told us the scaling for v-velocity.
  - – Y-momentum told us there is no pressure gradient normal to the plate.
  - – X-momentum then lost a viscous term (du/dx), and dp/dx written in terms of dU/dx with Bern. Eq.
- Computed BL thickness
- Transition for laminar and turbulent Reynolds numbers (5E5 is the engineering cutoff).
- BL grows with distance, Re grows with distance → becomes turbulent.
- Shear stress decreases with distance.
- Turbulent shear > laminar shear.
- RECALL HOW WE USE THESE FACTS, remember the 2 plate direction prob.

Class 29—External Flow

- Lift, Drag, Drag Coefficient.
- Drag is emphasized and is the net force in flow direction.
  - Pressure forces (form drag)
    - Flow separation
  - Viscous forces (friction drag)
  - Streamlining reduces pressure drag, but increases friction drag, but usually, pressure drag dominates.
- $F = C_d\rho v^2A/2$
  - A is the projected area, or the planview area
  - Use Table 11-1, 11-2 (given on exam if needed)
  - If v is unknown, guess laminar or turbulent → Cd → v → Re → confirm or adjust Cd. (Tables → only one try needed)
  - For small particles guess laminar, for big objects, guess turbulent.
- Terminal velocity follows from a force balance. Object falls under gravity until forces balance: buoyant, weight, drag

Class 30—Pumps

- Pump types: Positive displacement, dynamic.
- Dynamic = Centrifugal, Mixed Flow, Axial.
- bhp, efficiency. $\rho*g*Q*H/bhp$
- Pump performance curves (PPC): Head versus Flow
  - Shutoff head, free delivery flow rate, intermediate.
  - Individual pump whether alone, series or parallel operates on PPC.
  - System curve is system head versus flow.
  - Operating point where the SC and PPC intersect.
- Pump selection: size for required flow/head, maximize η, may have to overspecify pump (higher flow for given head)
- NPSH.
  - Specified for given pump.
  - Lookup P_{vap} for given flow. Flow parameters → P_2, v_2. Vary operation to ensure greater than NPSH. (Pressure drops prior to pump may lead to cavitation if $NPSH_{have} < NPSH_{need}$).

Class 31—Pump Scaling

- Series and Parallel
- Series, increase head for given flow.
  - PPC is sum (vertically) of individual pumps PPC
  - Don't operate beyond Free Delivery Flow for smallest pump, or shutoff/bypass that pump.
  - Can act as a head loss above the free delivery flow.
  - Know how the series pumps work with respect to a system demand curve
- Parallel, increase flow for given head.
  - PPC is sum (horizontally) of individual pumps PPC.
  - Don't operate beynod the shutoff head for the smallest pump, or bypass that pump.
  - Backflow above the shutoff head.
  - Each pump operates on its own PPC, but combined system operates on the compined PPC.
  - But the head over each pump is equal → provides a link (horizontal line on PPC) between individual and combined PPC
  - Again, know how this works on a system curve
- Scaling:
  - C_H, C_P, $\eta = f(C_Q, Re, \varepsilon/D) \sim f(C_Q)$. gH, bhp, $\eta = f(D, Q, \omega, \rho, \mu, \varepsilon)$
  - To scale, as usual, match these groups. Often, keeping one or several parameters const →simplifies the problem.
- Choose pumps based on value of N_{sp}
  - if you were given a set of requirements, know how to compute Nsp and choose a pump or turbine.

Class 32—Turbines

- Types: Hydraulic, wind, steam, gas, etc.
- Positive displacement, Dynamic
- Dynamic
  - Impulse (Pelton Wheel)
  - Reaction (like inverse pump)
- Pelton Wheel Analysed using momentum balance
- Reaction turbines:
  - Francis (Radial/mixed flow)
  - Kaplan (Axial flow)
- $\eta = 1/\eta_{pump}$
- Wind turbine
  - Work available, $\eta = C_p < 0.59$ (more like 0.45)
- Scaling laws are like pumps: C_H, C_Q, C_p, η
- Take C_P as independent parameter
- Turbine selection, based on N_{sp}.