ChE 374–Lecture 25–Integral Momentum Balance

- Integral Momentum Balance
  - Find Forces or Accelerations
- $\sum \vec{F} = \frac{\partial}{\partial t}\int_{CV} \rho \vec{v} dV + \int_{CS} \rho \vec{v}\vec{v} \cdot \vec{n} dA$.
  - For uniform properties:
    $\sum \vec{F} = \frac{d}{dt}(m\vec{v}) + (\sum \dot{m}\vec{v})_{out} - (\sum \dot{m}\vec{v})_{in}$
  - For uniform properties and Steady State:
    $\sum \vec{F} = (\sum \dot{m}\vec{v})_{out} - (\sum \dot{m}\vec{v})_{in}$
    * TAKE THIS EQUATION LITERALLY.
      · Vector components are positive in the positive coordinate direction.
      · $\dot{m} = \rho A|v|$ is a scalar quantity, always positive.
  - Note: $\dot{m}\vec{v}$ is momentum flow rate.
  - Note: $\rho\vec{v}\vec{v}$ is momentum flux.
  - Note: $\rho v_x \vec{v}$ is x-momentum flux.
- Choose a control volume
  - Not limited to the fluid
  - Choose it perpendicular to inlets and outlets
  - The momentum balance is WITH RESPECT TO THIS CV.
    * So velocities are RELATIVE to the control volume.
    * Also, $\dot{m}$ is also relative to the control volume.
- Forces are (1) BODY; (2) SURFACE; (3) OTHER
  - Body forces are gravity (mg).
  - Surface forces are Pressure (always normal to CV, and towards the CV); and viscous forces (usually neglect at inlets and outlets where the flows often cross the boundary).
  - Other forces are external, like bolts, the ground, and other anchoring forces, etc.
- Momentum flux correction factor corrects for nonuniform flow at inlets and outlets. Similar to kinetic energy correction factor.

  $\beta\overline{v} = \frac{1}{A}\int v^2 dA$. $\beta = 4/3$ for laminar
  $\alpha\overline{v} = \frac{1}{A}\int v^3 dA$. $\alpha = 2$ for laminar
- Example: Flow through a nozzle.
  - Restraining force is $F = A_1(P_1 + \rho v_1^2(1 - A_1/A_2)$, where (1) is inlet, (2) is outlet, and F is in the direction opposite the flow.
- Example: Flow deflected 90 degrees:
  - Restraining force is $F = \dot{m}v$, with force directed opposite the flow.
- Example: Flow deflected 180 degrees:
  - Restraining force is $F = 2\dot{m}v$, with force directed opposite the flow.
- Example: Flow deflected $\theta < 90$ degrees:

  $F_x = \dot{m}v(1 - \cos\theta)$, x is flow inlet direction, and F_x is opposite the flow direction.

  $F_y = \dot{m}v(\sin\theta)$, y is perpendicular to inlet direction, and F_y is with the direction of deflection.

Class 25 - Momentum Balance.

Where are we ?
- Fluid Statics
- Mass Balance
- Energy Balance
- * Momentum Balance.
  Chp 6.1-6.4 (linear Momentum)

- Velocity Profiles and Forces → Mom. Bal.
  - $F = ma = m\frac{dv}{dt} = \frac{dmv}{dt}$ = Rate of Momentum.
  - Derived Laminar pipe velocity using a Force (momentum) Balance.
    - Used a Differential Approach.
    - * - Here, Do Integral Approach, Then Discuss (Later Class) Differential Balances formally.

Goal : Find $\vec{F}$, $\vec{a}$, $\vec{v}$, Given forces, or $\vec{v}$, etc.

Reynolds Transport Theorem

$$\underbrace{\frac{dB_{sys}}{dt}} = \frac{\partial}{\partial t}\int \rho b \, d\mathcal{V} + \int_{cs} \rho b \vec{v}\cdot\vec{n}\, dA \quad ; \quad B = m\vec{v}, \quad b = \vec{v}$$

$$* \quad \sum \vec{F} = \frac{\partial}{\partial t}\int \rho \vec{v}\, d\mathcal{V} + \int_{cs} \rho \vec{v}\, \vec{v}\cdot\vec{n}\, dA$$

Uniform Properties :

$$* \quad \sum \vec{F} = \underbrace{\frac{d}{dt}(m\vec{v})}_{0 \text{ at S.S.}} + \left(\sum \dot{m}\vec{v}\right)_{out} - \left(\sum \dot{m}\vec{v}\right)_{in}$$

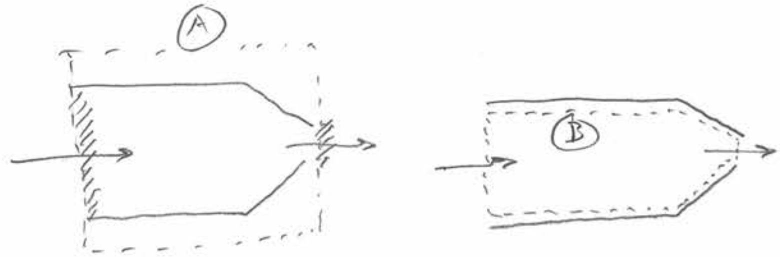
0 at S.S. ($\vec{v}$ here is $\vec{v}$ of C.V. $\neq \vec{v}_{out}, \vec{v}_{in}$)

- Vector Eq: → Split into x, y Components.

Note: $\dot{m}\vec{v}$ is momentum rate : $kg \cdot m/s^2$
$\rho\vec{v}\vec{v}$ is momentum flux : kg/ms^2

Control Volume

- we've implied The use of a C.V.
- Velocities are relative to the C.V. ; $V = V - V_{cv}$
- Be Smart:



Ⓐ → ignore The Details inside → easier

Ⓑ → must account for all forces of The nozzle, Fluid, → harder

- Choose C.V. $\perp$ to inlet / outlet.
- C.V. is not limited to Fluid
  - in Ⓐ, only have nonzero x-Forces in The Shaded region.

$\Sigma \vec{F}$ Term

- These are $\boxed{\text{external}}$ Forces
  1. Body Forces
  2. Surface Forces (Fluid)
  3. Other

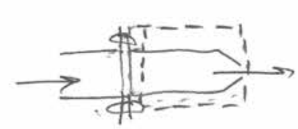
① Body Forces = Gravity $= m\vec{g} = -\rho V g = F_y$

② Surface Forces = Pressure
Viscous (Ignore, usually)

- $\vec{F}_p = -p\vec{n}$

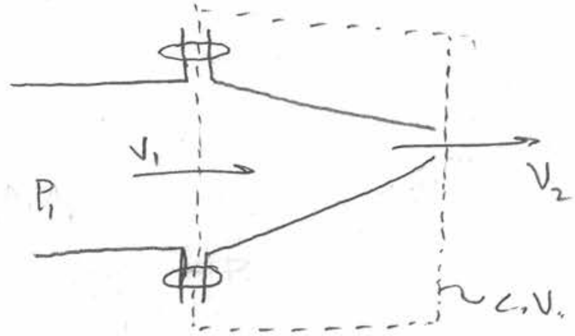
$F_{P,x} = P_1 A - P_2 A$
+ in + x Dir

③ Other
- Bolts, Ground, Anchoring forces, etc.
- $\vec{F}_o$ is the Force of These on The C.V.
- Careful of The Sign
- Ground exerts upward Force;



Bolts pull ←

Example 1



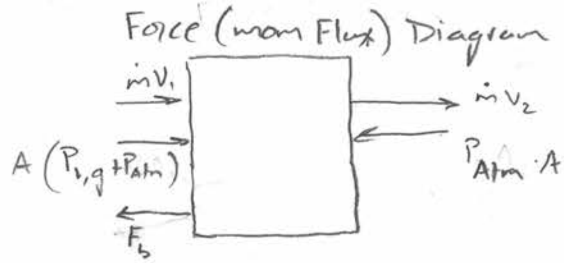
- Flow through Nozzle
- S.S.
- * Find Force on Bolts,
  Given P_1, V_1, A_1, A_2

Governing Eq.

$$\left(\sum F\right) = \left(\frac{dm\vec{v}}{dt}\right) + (\dot{m}\vec{v})_{out} - (\dot{m}\vec{v})_{in}$$

- $\dot{m} = \rho A |v|$
- Forces from P_1 Bolts

Force (mom Flux) Diagram



$$\left(-F_b + A(P_{1,g} + P_{Atm} - P_{Atm})\right) = (0) + \rho A_2 V_2^2 - \rho A_1 V_1^2$$

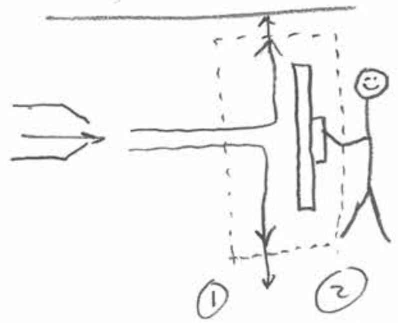
- $V_1 A_1 = V_2 A_2$

$$F_b = P_{1,g} \cdot A_1 + \rho A_1 V_1^2 - \rho A_2 \left(\frac{V_1 A_1}{A_2}\right)^2$$

* $$F_b = A_1\left[P_1 + \rho V_1^2\left(1 - \frac{A_1}{A_2}\right)\right]$$

Note: P_{atm} acts on all Surfaces
→ Cancels
→ Work in Gage Pressure

Example 2



- Flow turns, Find holding Force
- S.S.

y - Direction → Cancels

x - Direction

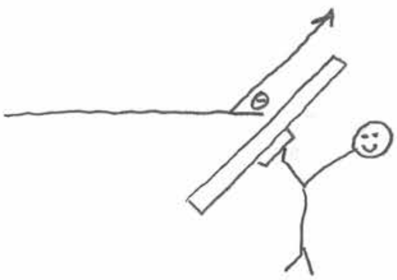
- Atm Cancels

$$\sum \vec{F} = \cancel{\frac{dm\vec{v}}{dt}} + \cancel{\dot{m}\vec{v}_{out}} - \dot{m}\vec{v}_{in}$$

$$-F_{arm} = -\dot{m} V_1$$

$$\boxed{F_{arm} = \dot{m} V_1}$$

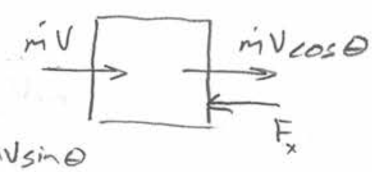
Example 3



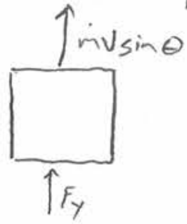
- Incline
- SS
- Find Force

$$\sum \vec{F} = \cancel{\frac{d\,m\vec{V}}{dt}} + (\dot{m}\vec{V})_o - (\dot{m}\vec{V})_i$$

X-momentum: $-F_x = \dot{m}V\cos\theta - \dot{m}V$

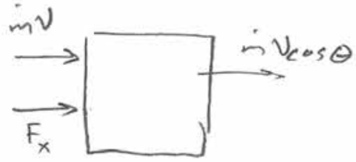


Y-momentum: $F_y = \dot{m}V\sin\theta$



$$* \begin{cases} F_x = \dot{m}V(1-\cos\theta) \\ F_y = \dot{m}V\sin\theta \\ \vec{F} = -F_x\vec{i} + F_y\vec{j} \end{cases}$$

Note: Could have written X- as:



$\rightarrow F_x = \dot{m}V\cos\theta - \dot{m}V$

$= \dot{m}V(\cos\theta - 1)$ is negative So force acts $\leftarrow$

If Draw F_x $\leftarrow$, and Sign in eqn implies $\leftarrow$, Then a Pos F_x is $\leftarrow$

If Draw F_x $\rightarrow$, and Sign in eqn implies $\rightarrow$, Then a Pos F_x is $\rightarrow$

If confused, Always Draw F_x in $+X$ Dir. Then $+F_x$ $\rightarrow$
$-F_x$ $\leftarrow$

Example 4

S.S.
X-only

$$\sum \vec{F} = \cancel{\frac{d\,m\vec{V}}{dt}} + (\dot{m}V)_o - (\dot{m}V)_i$$

$-F_{arm} = (-\dot{m}V) - (\dot{m}V)$

$\boxed{F_{arm} = 2\dot{m}V}$ = 2x  !

- See All Book Examples
- What if unsteady?
- Be careful about interpreting Thrust in Book:

$$\sum \cancel{\vec{F}_{ext}} = \frac{dm\vec{V}}{dt} + \dot{m}\vec{V}_{out} - (\dot{m}\vec{V}_{in})$$

0

$$\rightarrow \underbrace{\frac{dm\vec{V}}{dt}}_{} = (\dot{m}\vec{V})_{in} - (\dot{m}\vec{V})_{out}$$

Book calls This $\vec{F}_{thrust}$, but it is just The unsteady acceleration Term, Driven By The Rate of momentum Transfer.

- Know what to Do if not uniform

$\rightarrow$ Backup and integrate:

$$S.S. : \quad \sum \vec{F} = \int_{C.S.} \rho \vec{V} V_{out}\, dA - \int_{C.S.} \rho \vec{V} V_{in}\, dA$$