Chemical Engineering 374

Fluid Mechanics

Exam 2 Review

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Exam Review, By Content/Lectures

- Classes 13-21
- Chapter 5.6 Mechanical Energy
- Chapter 7.1-7.5 Dimensional Analysis
- Chapter 8.1-8.8 Pipe Flows
  - Laminar
  - Turbulent
  - Minor Losses
  - Single Pipelines
  - Pipe networks
  - Flow measurement

Class 13—Mechanical Energy

- Mechanical Energy Balance
  - Steady state $\left(\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2} + g\Delta z\right) = \frac{\dot{W}}{\dot{m}} - \frac{\dot{F}}{\dot{m}}$
  - Friction losses included but just given
  - ME changes due to shaft work and friction
  - What if have heat transfer?
  - Head form?
- KE correction factor α
  - Know this, but usually ignore, unless laminar
- Examples similar to Chp 8, where we compute the friction losses explicitly.

Class 14—Dimensional Analysis

- Dimensional Homogeneity
- Nondimensionalize equations: $t^*=t/t_{ref}$; $t=(t_{ref})(t^*)$
- Scaling
  - Terms in equations become product of [..](..), where [..] give size, (..) is O(1).
  - Can use this to find functional forms.
    - Heat equation characteristic time
    - Boundary layer thickness.
- Similarity—3 types (geometric, kinematic, dynamic)
  - Equivalent systems are similar
  - The Π groups need to be equal between model and full-scale

Class 15—Dimensional Analysis

- Find dimensionless groups (Π's)
  - 3 methods: governing equations, force ratios, **Π method**
- Π method is general (but can be cumbersome)
  - n parameters
  - j dimensions
  - k=n-j Π's
- Fluids, usually have 3 dimensions (m, s, kg)
- Can usually guess the Π's directly
  - Use all parameters
  - Helps to pick j=n-k repeating parameters that appear in all Π's
  - Re is common in fluids with viscous effects (friction, drag, etc.)

Class 16—Laminar Pipe Flow

- Pipe discussion
- More on the Re (physical intuition: force ratio, timescale ratio)
  - Re < 2300 is laminar; Re > 4000 is turbulent (transition in between)
  - 2300 is the number to remember as the laminar/turbulent cutoff
  - Most flows are turbulent
- Hydraulic diameter (for noncircular pipes) $D_h=4A_c/P_w$
- Entrance region
  - Fully developed flow takes time/space
  - Wall stress/friction/pressure drop is higher in entrance region.
- Derive the velocity profile
  - Force balance → ODE (pressure, wall friction) (BC: v=0 at wall, dv/dx=0 at CL)
  - 2 integrations → parabolic profile
  - dp/dx is constant
  - v_{avg} = ½ v_{max} (for circular pipes!)
  - f defined, f = 64/Re

$$f \equiv \frac{4\tau_w}{\rho v^2/2}$$

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Class 17—Turbulent Flow

- Darcy friction factor: $f = 8\tau_w/\rho v_{avg}^2$
  - $\Delta P_L/\rho = fLv^2/2D$
  - $f = 4 * f_{Fanning}$
- Velocity profile is flatter, with steep wall gradients → higher wall friction → higher friction than laminar.
  - Power law velocity profiles available
- Pipe roughness is important due to thin viscous wall layer.
- Params are $\Delta P/L$, ρ, μ, ε, D, v → 3 Π's → f(Re, ε/D)
  - Colbrook equation (implicit)
  - Haaland equation
  - Moody chart
  - Note simplifications: rough pipes at high Re → f is constant (read off Moody, or simple Colbrook)
  - Smooth pipes give a simpler Colbrook equation

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Class 18—Minor (fitting) Losses

- Losses from fittings, bends, valves, etc.
- In a pipe of a given size, use ΣK_L
- Table 8-4
- Based on the smaller of two diameters
- Use loss coefficient or equivalent lengh and add to pipe length.
- Don't forget expansions! K=1 (K=α).
- Contractions → vena contracta
- Valves increase friction to decrease flow rate for given pressure difference.

Class 19—Single Pipelines

$$\left(\frac{\Delta P}{\rho}+\frac{\Delta v^2}{2}+g\Delta z\right)=-\frac{fLv^2}{2D}-\frac{\Sigma K_L v^2}{2}+\frac{\dot{W}_s}{\dot{m}}$$

- Note well the mechanical energy balance equation above with shaft (pump) work, pipe losses, and minor losses.
- Three problem types.
  - Find P, find $\mathscr{V}$, find D
  - Some require iteration. Know how to do this to solve the three types.
    - Find $\mathscr{V}$: Guess f, v from E eqn., get Re, get f=f(Re, e/D), repeat.
    - Find D : Guess D, get Re, get f=f(Re,e/D), D from E. eqn, repeat.
  - System demand curve can turn a harder type into an easy type to "bypass" iteration.
- Examples given
- Economic pipe diameter (velocity), pipe size charts.

Class 20—Pipe Networks

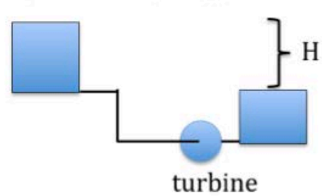
- 2 Key parameters: ΔP, $\mathscr{V}$
- Series Flow
  - $\Delta P_{tot} = \Sigma \Delta P_i$
  - Constant $\mathscr{V}$
- Parallel Flow
  - $\mathscr{V}_{tot} = \Sigma \mathscr{V}_i$
  - $\Delta P_i = \Delta P_j = \Delta P_k$ For pipes between the same two nodes
- Type 1 (find ΔP) and 2 (find $\mathscr{V}$) problems considered
- A system demand curve can help conceptually (and computationally)
- Can also set up and solve system of nonlinear equations
- More complex networks are the sum of the parts
  - $\Sigma Q_i=0$ at "nodes" (pipe junctions)
  - $\Sigma \Delta P_i=0$ around loops.
  - Like Kirchoff's laws for current flow (but nonlinear)

Class 21—Flow Measurement

- Flow meter types discussed
- Emphasis on Bernoulli types
  - Pitot, Pitot-static
  - Orifice meters
  - Nozzles
  - Venturi meters
- Rotameters
- Discharge coefficient correlations provided
  - Require iteration, but good guesses given low variation of C_d
  - Book provides average values (their values agree better with correlations at lower Re)
    - 0.61, 0.96, 0.98 for orifice, nozzle, venturi, respectively

$$v = C_d\sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

Water flows from a reservoir, into a **sharp-edged pipe** (K_L=0.5), through a couple of **90° miter bends** (K_L=1.1) to a lower reservoir as shown in the figure. The difference in the levels of the reservoirs is **H=100 m**. The water flows through a pipe of length **L=200 m** and relative roughness **ε/D=0.002**. The velocity in the pipe is **v=10 m/s**, giving a very **high Re**. If the power generated by the turbine is **W_t=4,437,500 J/s**, **Find the required pipe diameter**.



We have seen how turbulence consists of large eddies breaking into smaller eddies until the scales are small enough that viscosity dissipates the kinetic energy (mechanical energy) into internal energy (friction). A most remarkable property of turbulence was found by Kolmogorov using dimensional analysis. He argued that the small scales are determined only by the **kinematic viscosity ν (m^2/s)** and the rate that energy is fed to the small scales by the large scales—the **dissipation rate ε (m^2/s^3).**

(a) Using some combination of these two parameters, **form a length scale η (m)**, and a **time scale τ (s)** for the small scales (the Kolmogorov (or small eddy) scales).

(b) At high Re, the rate of energy transfer at large scales (big eddies breaking into small ones, etc.) does not depend on viscosity. **Write the dissipation rate ε as a combination of the large length scale L and the large scale velocity v.**

(c) Now, find η in a 10 cm diameter pipe (that is, L=D=10 cm) at Re = 5,000?

In the pipe network shown in the figure, the total flow rate is 15.
Find the flow rate and pressure drop through pipe c.
The system demand curves for each pipe INDIVIDUALLY are shown in the figure.
Show your work for full/partial credit.