ChE 374–Lecture 20–Pipe Networks

- As we often consider flow in single pipes, pipe networks are the rule not the exception.
- Series and Parallel Pipes.
- Series Flow
  - Total flow rate is constant by mass conservation.
  - Total pressure drop is that through each individual section.
    * Includes minor losses.
      · For expansions and contractions, losses "belong" to the smaller pipe.
  - Type I problem: Know D, L, Flow, $\rightarrow$ get ΔP_L.
    * Calculate Re, f, ΔP_L in each section and add upp the pressure losses in each.
  - Type II problem: Know ΔP_L, D, L, find $\dot{\mathcal{V}}$.
    * Produce a system demand curve for each pipe: ΔP_L vs. $\dot{\mathcal{V}}$.
    * $\dot{\mathcal{V}}$ is constant and $\Delta P_L = \sum_i \Delta P_{L,i}$.
    * Add the curves vertically to get $\dot{\mathcal{V}}_{tot}$ curve and read off $\dot{\mathcal{V}}_{tot}$ for given pressure loss.
    * Or, write equation $\Delta P_L = \sum_i \Delta P_{L,i}$ explicitly and solve nonlinear equations.
  - Type III or IV problems: Find D or L. This doesn't count: not unique.
  - Mathcad example for Type II problem.
- Parallel Pipe Networks.
  - Total flow rate is the sum of the flow rates in individual pipes.
  - Pressure drop is the same through each pipe connected to the same two junction points.
  - Type I problem: Flow rate known in a given pipe, ΔP_L unknown:
    * Compute ΔP_L in known pipe as usual, then with ΔP_L known in the other pipes, compute flow rate as a usual Type II problem.
  - Type II problem: Pressure drop known, compute flow rates.
    * Compute a type II problem in each pipe as usual.
  - Type I propblem: Total flow rate known, pressure drop unknown (hard).
    * Draw a system demand curve for each pipe. Add horizontally to get the total flow rate versus pressure drop curve. Then for known total flow, pressure drop is known and read off the flow rates through each pipe at that pressure drop. Or, once pressure drop and total flow are known, solve type II problem in each pipe.
    * Can also solve system of nonlinear equations, equating pressure drops through each pipe and the expression for pressure drop in terms of flow rate and total flow rate is the sum.
- Can devise complex networks, applying the above principles
  - Like Kirchoff's laws for electrical networks, but harder since we have nonlinear systems of equations.
    * Use a multi-dimensional version of Newton's method (ChE 541).

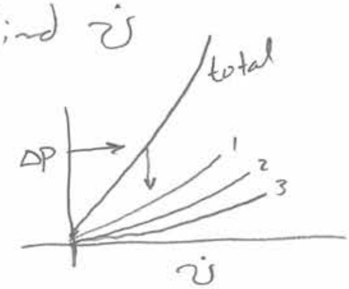
Lecture 20 — Pipe Networks

- Fluid Distribution systems
  - Air — Heating/Cooling
  - water — Cities
  - oil — refineries
- Series / Parallel
- Similar analysis to electrical Circuits

Series Flow

2 principles:

- $\dot{V}$ const by mass conservation
  $\dot{V}_1 = \dot{V}_2 = \dot{V}_3 = \dot{V}_4$
- $\Delta P_{tot} = \sum \Delta P_i$
- Expansions / Contractions $\rightarrow$ eval at the Smaller pipe.
- Type I : Know $L, D, \dot{V} \rightarrow$ Find ΔP
  Calc Re, f, ΔP in each Section $\rightarrow$ Sum
- Type II : Know ΔP, D, find $\dot{V}$
  - System Demand Curve

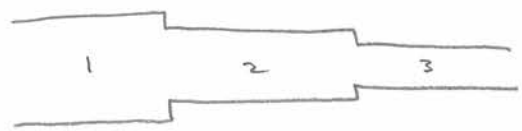


add: ↑

  - Read off $\dot{V}$ from ΔP_{tot} vs $\dot{V}$ curve, given ΔP_{tot}
  - OR: Solve $\Delta P_t = \sum_i \frac{L_i}{D_i} \frac{\rho V_i^2}{2} f_i$ with Colebrook for f_i
    $\rightarrow$ Solve nonlinear system of eqns.

- Type III : find D → Doesn't count, No unique D, Lots of pipes. (unless know all but one pipe, but then its not a network problem)

- Example : 3 pipes :



	L (m)	d (m)	ϵ (m)
1	100	0.05	0.00024
2	150	0.045	0.00012
3	80	0.04	0.0002

- Find $\dot{V}$ for $\Delta P_t = 320{,}000$ Pa
- Ignore Minor Loss

$$\Delta P_1 + \Delta P_2 + \Delta P_3 = \Delta P_t$$
$$\Delta P_1 = \rho f_1 L_1 v_1^2 / 2 D_1$$
$$\Delta P_2 = \text{"}$$
$$\Delta P_3 = \text{"}$$

$$V_i = \frac{\dot{V}_i}{A_i}$$
$$f_i = f_i(Re_i, \epsilon_i/D_i)$$
$$Re_i = \rho D_i v_i / \mu$$

Solve system with Mathcad: See attached

(3)

Mathcad - [Series_pipes.xmcd]

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$Pt := 320000 \qquad \rho := 998 \qquad \mu := 1.002 \cdot 10^{-3}$

$e1 := 0.00024 \qquad L1 := 100 \qquad D1 := 0.05$

$e2 := 0.00012 \qquad L2 := 150 \qquad D2 := 0.045$

$e3 := 0.0002 \qquad L3 := 80 \qquad D3 := 0.04$

Guesses

$f1 := 0.01$

$f2 := 0.01$

$f3 := 0.01$

$Q := 0.1$

Given

$$Pt = f1 \cdot \frac{L1}{D1} \cdot \frac{\rho}{2} \cdot \left(\frac{4 \cdot Q}{\pi \cdot D1^2}\right)^2 + f2 \cdot \frac{L2}{D2} \cdot \frac{\rho}{2} \cdot \left(\frac{4 \cdot Q}{\pi \cdot D2^2}\right)^2 + f3 \cdot \frac{L3}{D3} \cdot \frac{\rho}{2} \cdot \underbrace{\left(\frac{4 \cdot Q}{\pi \cdot D3^2}\right)}_{V}^2$$

$$\frac{1}{\sqrt{f1}} + 2 \cdot \log\left[\frac{e1}{D1 \cdot 3.7} + \frac{2.51}{\left(\frac{\rho \cdot D1}{\mu} \cdot \frac{4 \cdot Q}{\pi \cdot D1^2}\right) \cdot \sqrt{f1}}\right] = 0$$

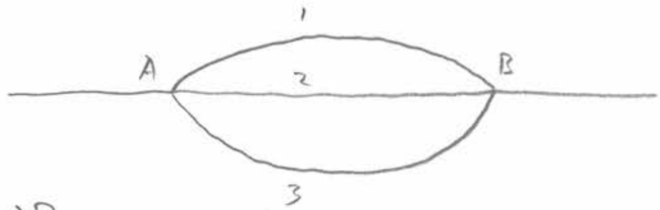
$$\frac{1}{\sqrt{f2}} + 2 \cdot \log\left[\frac{e2}{D2 \cdot 3.7} + \frac{2.51}{\left(\frac{\rho \cdot D2}{\mu} \cdot \frac{4 \cdot Q}{\pi \cdot D2^2}\right) \cdot \sqrt{f2}}\right] = 0$$

$$\frac{1}{\sqrt{f3}} + 2 \cdot \log\left[\frac{e3}{D3 \cdot 3.7} + \frac{2.51}{\underbrace{\left(\frac{\rho \cdot D3}{\mu} \cdot \frac{4 \cdot Q}{\pi \cdot D3^2}\right)}_{Re} \cdot \sqrt{f3}}\right] = 0$$

$$\begin{pmatrix} Q \\ f1 \\ f2 \\ f3 \end{pmatrix} := \text{Find}(Q, f1, f2, f3)$$

$$\begin{pmatrix} Q \\ f1 \\ f2 \\ f3 \end{pmatrix} = \begin{pmatrix} 2.64074 \times 10^{-3} \\ 0.03141 \\ 0.02716 \\ 0.03148 \end{pmatrix}$$

Parallel pipe Networks.

- Total flow rate is The Sum of Flow rates in individual pipes.
- 

$\Delta P_1 = \Delta P_2 = \Delta P_3$ Since P_A is same for pipes 1, 2, 3.
P_B is same for pipes 1, 2, 3

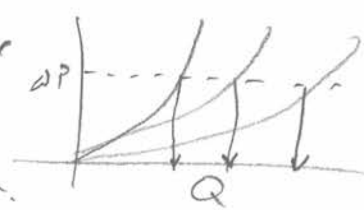
(easy) • Type I: flow rate Known ΔP unknown
(1 pipe)

- Compute ΔP in known pipe as usual
- Then w/ ΔP, Compute flow rates in other pipes as a Type II problem.

(easy) • Type II: ΔP Known, flow rates unknow.

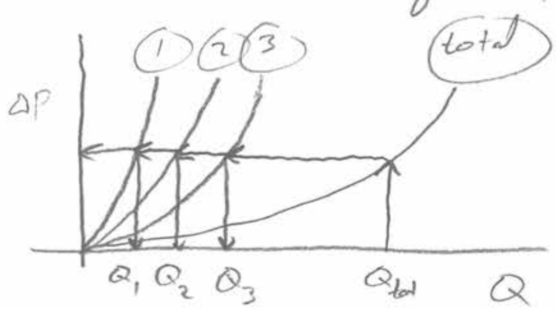
- Compute Type II problem in each pipe → Q
- $Q_t = \sum_i Q_i$

– System Demand Curve for each pipe

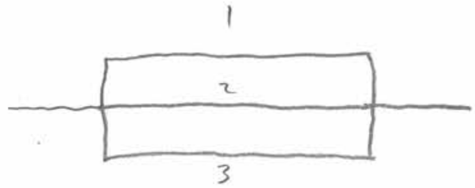


(harder) • Type I: Total flow rate Known, ΔP unknown.

– System Demand Curve for each pipe.



or ...



(1) $\Delta P_1 = \Delta P_2$

(2) $\Delta P_1 = \Delta P_3$

$\Bigg\}$ imply $\Delta P_2 = \Delta P_3$

(3) $\Delta P_1 = f_1 \frac{L_1}{D_1} \frac{\rho V_1^2}{2}$

(4) $\Delta P_2 = f_2 \frac{L_2}{D_2} \frac{\rho V_2^2}{2}$ use $V_i = \frac{Q_i}{\frac{\pi}{4} D_i^2}$

(5) $\Delta P_3 = f_3 \frac{L_2}{D_2} \frac{\rho V_3^2}{2}$ use $f_i = f_i\left(Re_i, \frac{\varepsilon_i}{D_i}\right)$ w/ $Re_i = \frac{\rho D_i}{\mu} \frac{Q_i}{\frac{\pi}{4} D_i^2}$

(6) $Q_t = Q_1 + Q_2 + Q_3$

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6 eqns in unknowns ΔP_1 ΔP_2 ΔP_3, Q_1, Q_2, Q_3

or:

$$\left.\begin{aligned} \frac{f_1 L_1}{2 D_1} \rho V_1^2 &= f_2 \frac{L_2}{D_2} \frac{\rho V_2^2}{2} \\ \frac{f_1 L_1}{2 D_1} \rho V_1^2 &= f_3 \frac{L_3}{D_3} \frac{\rho V_3^2}{2} \\ Q_t &= Q_1 + Q_2 + Q_3 \end{aligned}\right\} \rightarrow \text{3 eqns in 3 unknowns } Q_1, Q_2, Q_3$$

Then, Given Q's Solve for $\Delta P_1 = \Delta P_2 = \Delta P_3 = f_1 \frac{L_1}{D_1} \frac{\rho V_1^2}{2}$ as a Type 1 problem.

Can easily Develop more Complex networks

Apply principles To get an equation System.

1: Conserve mass : Series : Q = Const
Parallel : $Q_t = \sum Q_i$ bet. nodes.

2: ΔP : Series : $\Delta P_t = \sum \Delta P_i$
Parallel : ΔP = Same bet branches of 2 nodes

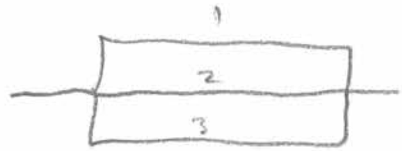
· at junctions $\sum Q_i = 0$
· at closed loops $\sum \Delta P = 0$

Like Kirchhoff's laws but more Complex because nonlinear.

- Multi-D Version of Newton's method. M
- Many Canned packages
- CHE 541 - .

Example

As before, but parallel not series.



	ϵ_1	L	D
1	0.00024	100	0.05
2	0.00012	150	0.045
3	0.0002	80	0.04

$\Delta P = ?$ $\quad \dot{Q} = 0.0133\ m^3/s.$

$\boxed{\text{See Mathcad.}}$

Mathcad - [Parallel_pipes.xmcd]

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$Qt := 0.01333 \qquad \rho := 998 \qquad \mu := 1.002\cdot 10^{-3}$

$e1 := 0.00024 \qquad L1 := 100 \qquad D1 := 0.05$

$e2 := 0.00012 \qquad L2 := 150 \qquad D2 := 0.045$

$e3 := 0.0002 \qquad L3 := 80 \qquad D3 := 0.04$

Guesses

$f1 := 0.01 \qquad Q1 := 0.004$

$f2 := 0.01 \qquad Q2 := 0.004$

$f3 := 0.01 \qquad Q3 := Qt - Q1 - Q2$

Given

$$f1\cdot\frac{L1}{D1}\cdot\frac{\rho}{2}\cdot\left(\frac{4\cdot Q1}{\pi\cdot D1^2}\right)^2 = f2\cdot\frac{L2}{D2}\cdot\frac{\rho}{2}\cdot\left(\frac{4\cdot Q2}{\pi\cdot D2^2}\right)^2$$

$$f1\cdot\frac{L1}{D1}\cdot\frac{\rho}{2}\cdot\left(\frac{4\cdot Q1}{\pi\cdot D1^2}\right)^2 = f3\cdot\frac{L3}{D3}\cdot\frac{\rho}{2}\cdot\left(\frac{4\cdot Q3}{\pi\cdot D3^2}\right)^2$$

$$\frac{1}{\sqrt{f1}} + 2\cdot\log\left[\frac{e1}{D1\cdot 3.7} + \frac{2.51}{\left(\frac{\rho\cdot D1}{\mu}\cdot\frac{4\cdot Q1}{\pi\cdot D1^2}\right)\cdot\sqrt{f1}}\right] = 0$$

$$\frac{1}{\sqrt{f2}} + 2\cdot\log\left[\frac{e2}{D2\cdot 3.7} + \frac{2.51}{\left(\frac{\rho\cdot D2}{\mu}\cdot\frac{4\cdot Q2}{\pi\cdot D2^2}\right)\cdot\sqrt{f2}}\right] = 0$$

$$\frac{1}{\sqrt{f3}} + 2\cdot\log\left[\frac{e3}{D3\cdot 3.7} + \frac{2.51}{\left(\frac{\rho\cdot D3}{\mu}\cdot\frac{4\cdot Q3}{\pi\cdot D3^2}\right)\cdot\sqrt{f3}}\right] = 0$$

$$Qt = Q1 + Q2 + Q3$$

$$\begin{pmatrix} f1 \\ f2 \\ f3 \\ Q1 \\ Q2 \\ Q3 \end{pmatrix} := \mathrm{Find}(f1, f2, f3, Q1, Q2, Q3) \qquad \begin{pmatrix} f1 \\ f2 \\ f3 \\ Q1 \\ Q2 \\ Q3 \end{pmatrix} = \begin{pmatrix} 0.030663 \\ 0.026613 \\ 0.03118 \\ 5.775203\times 10^{-3} \\ 3.889447\times 10^{-3} \\ 3.66535\times 10^{-3} \end{pmatrix}$$

$$\Delta P := f1\cdot\frac{L1}{D1}\cdot\frac{\rho}{2}\cdot\left(\frac{4\cdot Q1}{\pi\cdot D1^2}\right)^2 \qquad \Delta P = 2.647\times 10^5$$