Chemical Engineering 374

Fluid Mechanics

Single Pipelines

Pipelines with friction losses

$$\left(\frac{\Delta P}{\rho}+\frac{\Delta v^2}{2}+g\Delta z\right)=\frac{\dot{W}_u}{\dot{m}}-\frac{\dot{F}}{\dot{m}}$$

Ignoring the Work Term:

$$\left(\frac{\Delta P}{\rho}+\frac{\Delta v^2}{2}+g\Delta z\right)=\underbrace{-\frac{fLv^2}{2D}-\frac{Kv^2}{2}}_{-F(=-\dot{F}/\dot{m})}$$

One equation in One Unknown

Pick Two Points Carefully

Problem Types

Type	Pressure Drop	Flow Rate	Diameter	Length
I	**?**	X	X	X
II	X	**?**	X	X
III	X	X	**?**	X
IV	X	X	X	**?**

- Types I and IV are easy.
- Types II, III require iteration
- Parameters: ΔP, L, D, V, μ, ρ, ε
- Equations:
  - ① Re = $\rho Dv/\mu$
  - ② $f=f(Re, \varepsilon/D)$
  - ③ $\Delta P = fL\rho v^2/2D$

Problems I, II, III from Swamee-Jain Relations

See Book Page 380

Type I Procedure

Type	Pressure Drop	Flow Rate	Diameter	Length
I	**?**	X	X	X
II	X	**?**	X	X
III	X	X	**?**	X
IV	X	X	X	**?**

① Re = $\rho Dv/\mu$

② $f=f(Re, \varepsilon/D)$

③ $\Delta P = fL\rho v^2/2D$

- **Find Pressure Drop**
- Solve Re from (1)
- Solve f from Colbrook Eqn. (2) etc.
- Solve ΔP from (3)

Type II Procedure

Type	Pressure Drop	Flow Rate	Diameter	Length
I	?	X	X	X
II	X	?	X	X
III	X	X	?	X
IV	X	X	X	?

① $Re = \rho D v/\mu$

② $f=f(Re, \varepsilon/D)$

③ $\Delta P = fL\rho v^2/2D$

- **Find Flow Rate (velocity)**
- Implicit: v appears in (1), (2), (3)
- Iterate (several ways)

1. Guess f as fully turbulent (where Moody levels off)
2. v from (3)
3. Re from (1)
4. f from (2)
5. Repeat to convergence

OR

1. Guess Re
2. Compute f from (2)
3. v from (3)
4. Re from (1)
5. Repeate

Type III Procedure

Type	Pressure Drop	Flow Rate	Diameter	Length
I	?	X	X	X
II	X	?	X	X
III	X	X	?	X
IV	X	X	X	?

① $Re = \rho D v/\mu$

② $f=f(Re, \varepsilon/D)$

③ $\Delta P = fL\rho v^2/2D$

- **Find Pipe Diameter**
- Implicit: D appears in (1), (2), (3)
- Iterate (several ways):

1. Guess D
2. Re from (1)
3. f from (2)
4. D from 3
5. Repeate

Type IV Procedure

Type	Pressure Drop	Flow Rate	Diameter	Length
I	**?**	X	X	X
II	X	**?**	X	X
III	X	X	**?**	X
IV	X	X	X	**?**

① Re = $\rho Dv/\mu$
② f=f(Re, ε/D)
③ $\Delta P = fL\rho v^2/2D$

- **Find Pipe Length**
- Solve (1), for Re
- Solve (2) for f
- Solve (3) for L

- The iterative methods use a fixed point procedure where the output of the last step is the input of the first step.
  - Just repeate until the guessed variable stops changing
  - May not converge.
- Can make two guesses, then compute the two resulting variables, then interpolate the guess to to the desired result
  - Type 3 example
    - Guess D → solve for ΔP,
    - Guess another D → solve for ΔP
    - Linear interpolation of D to desired ΔP between two known points

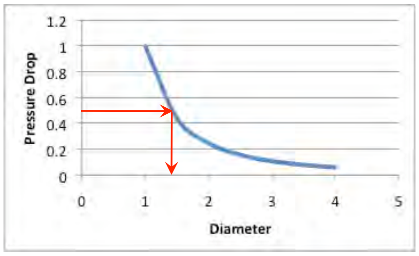
$$D_{new} = D_1 + (\Delta P_{want} - \Delta P_1)\frac{D_1 - D_2}{\Delta P_1 - \Delta P_2}$$

System Demand Curve

- An iterative approach amounts to solving the problem many times.
  - Can instead just solve a simpler problem for all values of the parameter and make a plot.
  - Then read off the solution from the plot.
- System Demand Curve
- Type 3 example: plot ΔP for many values of unknown D.
  - Guess many values of D,
  - Compute Re from (1)
  - Compute f from (2)
  - Compute ΔP from (3)

Do for each D

- Interpolate solution from result
- Or read off the plot
- Alternatively
  - Solve system of nonlinear equations
    - Excel Solver (or goal seek)
    - Mathcad solve block



What about power and head loss?

- If given power instead of pressure drop, just relate:
  - Power = $\dot{\mathcal{V}}\Delta P$
  - Can replace ΔP in (3) with $W_u/\dot{\mathcal{V}}$
- Flow rate is related to velocity as
  - $\dot{\mathcal{V}} = A*v = \pi/4 * D^2 * v$
- $h_L = \Delta P_L / \rho g$
- If have fittings, replace (3) with

$$\Delta P = \sum_i \frac{f_i L_i \rho v_i^2}{2D_i} + \sum_j \frac{K_j \rho v_j^2}{2}$$

Example 1

- Water at 60 °F → ρ = 62.36 lbm/ft^3, μ = 7.536E-4 lbm/ft*s
- D = 2 in
- ε = 0.000084 in
- $\mathcal{V}$ = 0.2 ft^3/s
- L = 200 ft
- Find ΔP, h$_L$, Power
- TYPE 1 PROBLEM
- Solve Re, then f, then ΔP, then h$_L$, then Power
- Re = ρDv/m
  - v =4$\mathcal{V}$/πD^2 =9.1673 ft/s
  - Re = 126310
- Colbrook → f = 0.0174 (solve on calculator)
- ΔP = fLρv^2/2D = 54713 lbm/ft*s^2 =11.81 psi
- h$_L$ =ΔP/ρg = 27.3 ft
- Power = $\mathcal{V}\Delta$P = 0.618 HP = 461 W
- If on an incline, ΔP = fLρv^2/2D → ΔP - ρgLsinθ = fLρv^2/2D
  - The combined term is the loss due to friction, → h$_L$ is the head loss from friction, and power is power to overcome the frictional losses. The total power needs to account for the elevation change as well (higher or lower power)

Example 2

- Air at 1 atm, 35 °C → ρ = 1.145 kg/m^3, μ = 1.895E-5 kg/m*s
  - L=150 m
  - ε = 0 (smooth)
  - $\mathcal{V}$ = 0.35 m^3/s
  - h$_L$ = 20 m (max)
  - **Find D**
- Type 3 problem
- Guess and check
  - D → Re → f → h$_L$ → repeate
- System demand curve
  - **See Matlab, Excel solutions**
  - Specify D, compute head loss: D→ Re → f → h$_L$
  - Solve the Colbrook equation with Newton's method
    - Use Haaland Equation as an initial guess.

F(x) =0 → solve for x
Write Colbrook Eqn. in this form

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

$$F(x) = \frac{1}{x} + 2\log\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\,x}\right) = 0$$

$$x = x_0 - \frac{F(x_0)}{\frac{dF}{dx}\big|_{x=x_0}}$$

Example 3

- Air at 1 atm, 35 ^oC → ρ = 1.145 kg/m^3, μ = 1.895E-5 kg/m*s
  - L=300 m
  - ε = 0 (smooth)
  - D=0.267 m
  - h_L = 20 m (max)
  - **Find $\mathcal{V}$**
- Type 2 problem
- Guess and check by hand
  - Re → f → v from (3) → Re from (1) → repeat
- Re = 100000 → 71068.5 → 68543 →68273 → 68244 → converged
- $\mathcal{V} = Av = \pi D^2 v/4 = 0.237\ m^3/s$
- Or, do system demand curve as before.
- Or, solve system of equations (see mathcad)

Economic Pipe Diameter and Pipe Sizes

- See attached book section from de Nevers (1991), Fluid Mechanics for Chemical Engineers
- Pipe costs, pump costs, maintenance costs.
- Key result: for water, the economic velocity is 6 ft/s = 1.88 m/s.
  - For a given flow rate, pick the diameter to give 6 ft/s.
- See attached table of pipe sizes.
  - (Just because it's a half inch pipe doesn't mean is 0.5 inches)

of the $\mathcal{F}$'s from one reservoir to another with the $g\,\Delta z$ term between the same two reservoirs. When the proper set of flows has been chosen, these will all agree. For this three-branch, one-node example, the trial-and-error method is quite easy (Prob. 6.60). For more complex examples, it is not. A widely used systematic procedure for solving this type of system was developed by Cross [13]. Computer programs are available to carry out that solution [14].

6.13 ECONOMIC PIPE DIAMETER

From the foregoing we can easily calculate the flow rate, given the pipe diameter and pressure drop, or calculate the pipe diameter, given the flow rate and pressure drop, and so forth. A much more interesting question is, Given the flow rate, what size of pipe should we select? It is possible that the choice is dictated by aesthetics; e.g., the pipe goes through a lobby, and we want it to be the same size as other exposed pipes in the lobby. Or the choice may be dictated by the supply; e.g., we have on hand a large amount of surplus 4-in pipe which we want to use. Most often the choice is based on economics; the engineer is asked to make the most economical selections, all things considered.

For economic analysis we must consider two possibilities:

1. The fluid is available at a high pressure and eventually will be throttled to a low pressure, so the energy needed to overcome friction losses may come from the available pressure drop.
2. The fluid is not available at a high pressure, so a pump or compressor is needed to overcome the effects of fluid friction.

The first is simple: We select the smallest size of pipe which will carry the required flow with the available pressure drop. Example 6.5 is that case.

If the effects of friction must be overcome by a pump or compressor, then the total annual costs of the pump pipeline system are the following:

1. Power to run the pump
2. Maintenance charges on pump and line
3. Capital-cost charges for both line and pump

How these change with increasing line size is sketched in Fig. 6.20. The figure indicates the following:

1. The larger the pipe diameter, the greater the capital charges. The cost of pipeline is roughly proportional to the pipe diameter; bigger pipes cost more to buy, require more expensive supports, take longer to install, etc. The cost of the pump is proportional to the cost of the pipe and is included in it.
2. The maintenance cost is not affected much by pipe size.

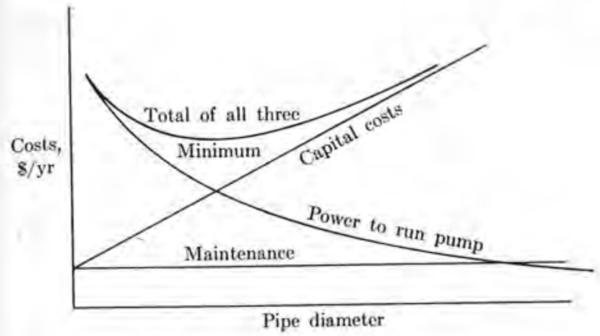


FIGURE 6.20

3. The pumping cost goes down rapidly as the pipe size goes up. The pumping cost is proportional to the pressure drop (see Example 6.3), which for turbulent flow is proportional to the velocity to the 1.8 to 2.0 power divided by the diameter. The velocity (for constant flow rate) is proportional to the reciprocal of the square of the diameter, so the pumping cost is proportional to the reciprocal of the diameter to the 4.6 to 5 power.

As Fig. 6.20 shows, the sum of these has a rather sharp minimum. This minimum occurs at the economic pipe diameter. Recognize here that we are taking the sum of a power cost during some finite period, e.g., a year, and the annual charge for owning the pipeline and the pump, whose lifetime will be many years. There are a variety of sophisticated ways of doing this, treated in books on plant design [15]. Here we consider the *simplest possible* kind of economic analysis:

$$\text{Purchase price} = \text{PP} \cdot \text{pipe diameter} \cdot \text{pipe length} \tag{6.39}$$

where the purchase price is what we would have to pay a contractor for both supplies and labor to build the complete pipeline and pump for us and PP is a constant with dimensions \$/[inch (of diameter) · ft (of length)].

$$\text{Annual capital charge} = \text{CC} \cdot \text{purchase price} \tag{6.40}$$

where capital charge (CC) is a constant, with dimension (1/year) and

$$\text{Annual pumping cost} = \text{PC} \cdot \text{pump power} \tag{6.41}$$

where pumping cost (PC) is a constant with dimensions \$/[hp · year].

As shown in Fig. 6.20, the maintenance cost is practically independent of the pipe diameter, so we do not include it in the analysis. We then wish to find the minimum of

$$\text{Total annual cost} = \text{PC} \cdot \text{Po} + \text{CC} \cdot \text{PP} \cdot \text{diameter} \cdot \text{length} \tag{6.42}$$

Assuming that the pipe is horizontal, we may apply Bernoulli's equation from the pump inlet, point 1, to the pipe outlet, point 2, and see that there is no change in elevation or velocity. We assume that the pressure at the pump inlet is the same as the pressure at the pipe outlet; i.e., the pump has to overcome only the effects of friction. Then from Eq. 6.16 we have

$$\frac{-dW_{\text{a.o.}}}{dm} = \mathscr{F} = 2f\,\frac{\Delta x}{D}\,V^2 \tag{6.43}$$

$$\text{Po} = \frac{-dW_{\text{a.o.}}}{dm}\,\dot{m} = 2f\,\frac{\Delta x}{D}\,V^2\dot{m} \tag{6.44}$$

but we have

$$V = \frac{\dot{m}}{\rho(\pi/4)D^2} \tag{6.45}$$

and therefore

$$\text{Po} = \frac{\dot{m}^3 2f\,\Delta x\,(4/\pi)^2}{\rho^2 D^5} \tag{6.46}$$

Substituting Eq. 6.43 and the cost of the pipe in Eq. 6.39, we find

$$\text{Total annual cost} = \text{PC}\cdot\dot{m}^3 2f\,\Delta x\left(\frac{4}{\pi}\right)^2\frac{1}{\rho^2}\cdot\frac{1}{D^5} + \text{CC}\cdot\Delta x\cdot\text{PP}\cdot D \tag{6.47}$$

We now differentiate the total annual cost with respect to diameter D and set the derivative equal to zero:

$$0 = \frac{d(\text{cost})}{dD} = \text{PC}\cdot\dot{m}^3 2f\,\Delta x\left(\frac{4}{\pi}\right)^2\frac{1}{\rho^2}\cdot\frac{-5}{D^6} + \text{CC}\cdot\Delta x\cdot\text{PP} \tag{6.48}$$

Solving for D_{econ}, we find

$$D_{\text{econ}} = \left[\frac{10\cdot\text{PC}\cdot\dot{m}^3 f(4/\pi)^2(1/\rho^2)}{\text{CC}\cdot\text{PP}}\right]^{1/6} \tag{6.49}$$

This equation shows that the economic pipe diameter is independent of how long the pipe is. This should be no surprise: Both the pumping and capital costs are proportional to the pipe length. The equation also shows that the economic diameter is proportional to the friction factor to the one-sixth power; hence, we can use a rough estimate of the friction factor and make very little error.

Example 6.17. We wish to transport 200 gal/min of water 5000 ft in a horizontal, schedule 40, carbon-steel pipe. We will install a pump to overcome the friction loss. Given the economic data shown below, what is the economic pipe diameter?

$$\text{PC} = \frac{\$270}{\text{hp}\cdot\text{yr}} \qquad \text{PP} = \frac{\$2}{\text{in of diameter}\cdot\text{ft of length}} \qquad \text{CC} = \frac{0.40}{\text{yr}}$$

First we guess that the pipe will have an inside diameter of 3 in. Then from Table 6.2 we have $\varepsilon/D = 0.0018/3 = 0.0006$. The friction factor will probably be about 0.0042. The mass flow rate is 200 gal/min · 8.33 lbm/gal = 1666 lbm/min. Substituting these and the values of PC, CC, and PP in Eq. 6.49 produces

$$D_{\text{econ}} = \left[\frac{\dfrac{\$270}{\text{hp}\cdot\text{yr}}\cdot\left(\dfrac{1666\text{ lbm}}{\text{min}}\right)^3\cdot 10\cdot 0.0042\cdot\left(\dfrac{4}{\pi}\right)^2\cdot\left(\dfrac{\text{ft}^3}{62.3\text{ lbm}}\right)^2}{0.4/\text{yr}\cdot\$2/(\text{in}\cdot\text{ft})}\right]^{1/6}$$

$$\cdot\left(\frac{\text{hp}\cdot\text{min}}{3.3\times10^4\text{ ft}\cdot\text{lbf}}\cdot\frac{\text{lbf}\cdot\text{s}^2}{32.2\text{ lbm}\cdot\text{ft}}\cdot\frac{\text{min}^2}{3600\text{ s}^2}\cdot\frac{\text{ft}}{12\text{ in}}\right)^{1/6}$$

$$= (5.95\times10^{-4}\text{ ft}^6)^{1/6} = 0.290\text{ ft} = 3.48\text{ in} = 0.088\text{ m}$$

Because of the approximate nature of the economic data used, a 4-in pipe would probably be selected. It would be appropriate to check the assumed friction factor (Prob. 6.62). ■

Because calculations such as these are long and tedious, companies that install many pipelines have solved the problem for a large number of cases and have summarized the results in convenient form. The most popular method is to calculate the economic velocity:

$$\text{Economic velocity} = \frac{\text{volumetric flow rate}}{(\pi/4)(\text{economic diameter})^2} \tag{6.50}$$

Substituting for the economic diameter from Eq. 6.49, we find

$$V_{\text{econ}} = \frac{\dot{m}/\rho}{\dot{m}(1/\rho^{2/3})f^{1/3}\cdot\text{constant}} = \text{constant}\cdot\frac{1}{f^{1/3}\rho^{1/3}} \tag{6.51}$$

This equation says that for a given set of cost data the economic velocity is independent of the mass flow handled and dependent on only the fluid density and the friction factor. More thorough analyses and far more complicated cost equations lead to substantially the same conclusion. For example, for schedule 40 carbon-steel pipe, Boucher and Alves [16] give the data shown in Table 6.4.

The table refers to turbulent flow only. For laminar flow, the value of f goes up quite rapidly as the viscosity increases, making the economic velocity go down. Oil companies spend more money pumping viscous liquids (crude oils, asphalt, heating oils, etc.) than do any other companies; therefore they have made up the most convenient economic-velocity plots for laminar flow.

TABLE 6.4
Economic velocity for schedule 40, carbon-steel pipe

Fluid density, lbm/ft	Economic velocity, ft/s
100	5.1
50	6.2
10	10.1
1	19.5
0.1	39.0
0.01	78.0

Figure 6.21 shows such a plot. It can be used to rapidly select the economic pipe diameter for laminar flow, subject to the restriction that the economic data on the line to be installed must be the same as those shown on the plot. Figure 6.21 has nomenclature similar to that of Fig. 6.13, and the comments on the latter are applicable here. Figure 6.21 also shows the economic diameter for turbulent flow.

Why does App. A.4 show the velocity in feet per second for all the water flows given? From Table 6.4 and Fig. 6.21 we can see that for water (which is almost always in turbulent flow in industrial equipment) an economic velocity is almost always about 6 ft/s. Thus, working engineers often simply select pipe sizes for water or similar fluids by looking at App. A.4 for the pipe size which gives a velocity of about 6 ft/s (2 m/s).

Table 6.4 and Fig. 6.21 are for one set of costs; for other costs the results are different. However, because of the $\frac{1}{6}$ factor in Eq. 6.49, the different costs change the economic diameter very little (see Prob. 6.66).

6.14 FLOW AROUND SUBMERGED OBJECTS

The flow around a submerged object is generally more complicated than the flow in a straight pipe or channel, because it is two- or three-dimensional. To understand the *details* of the flow around any submerged object, we must first take up the subjects of potential flow and the boundary layer, which we do in Chaps. 10 and 11.

Frequently we are not interested in the details of the flow but only in the practical problem of predicting the force on a body due to the flow of fluid around it. For example, the airplane designer wants to know the "air resistance" of the plane to select the right engine, the submarine designer wants to know the "water resistance" to determine how fast the submarine can go, and the designer of a chimney wants to know the maximum wind force on it to decide how much bracing is needed. These forces are now all called *drag forces*, following aeronautical engineering terminology. By using experimental data on such flows we can treat the problems as if they were one-dimensional.

Probably the first systematic investigation of drag forces was undertaken by Isaac Newton [17], who dropped hollow spheres from the inside of the dome of St. Paul's Cathedral in London and measured their rate of fall. He calculated that the drag force on a sphere should be given by

$$\text{Drag force} = F = \pi r^2 \rho_{\text{air}} \frac{V^2}{2} \tag{6.52}$$

Subsequent workers found that this equation had to be modified by introducing a coefficient, which we call the *drag coefficient* C_d. This coefficient is not a constant equal to 1, as Newton believed, but varies with varying conditions, as we will see. Introducing it and dividing both sides of Eq. 6.52 by the cross-sectional area of the sphere, we find

$$\frac{F}{A} = C_d \rho \frac{V^2}{2} \tag{6.53}$$

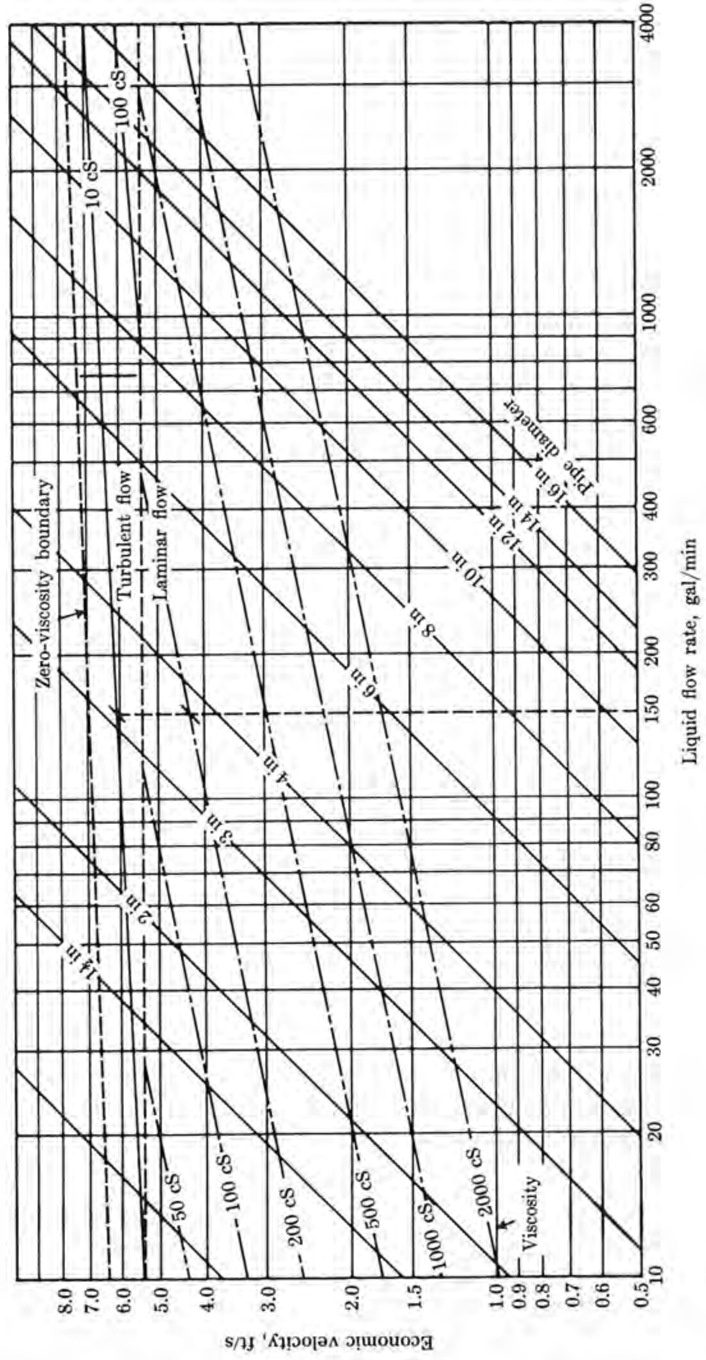
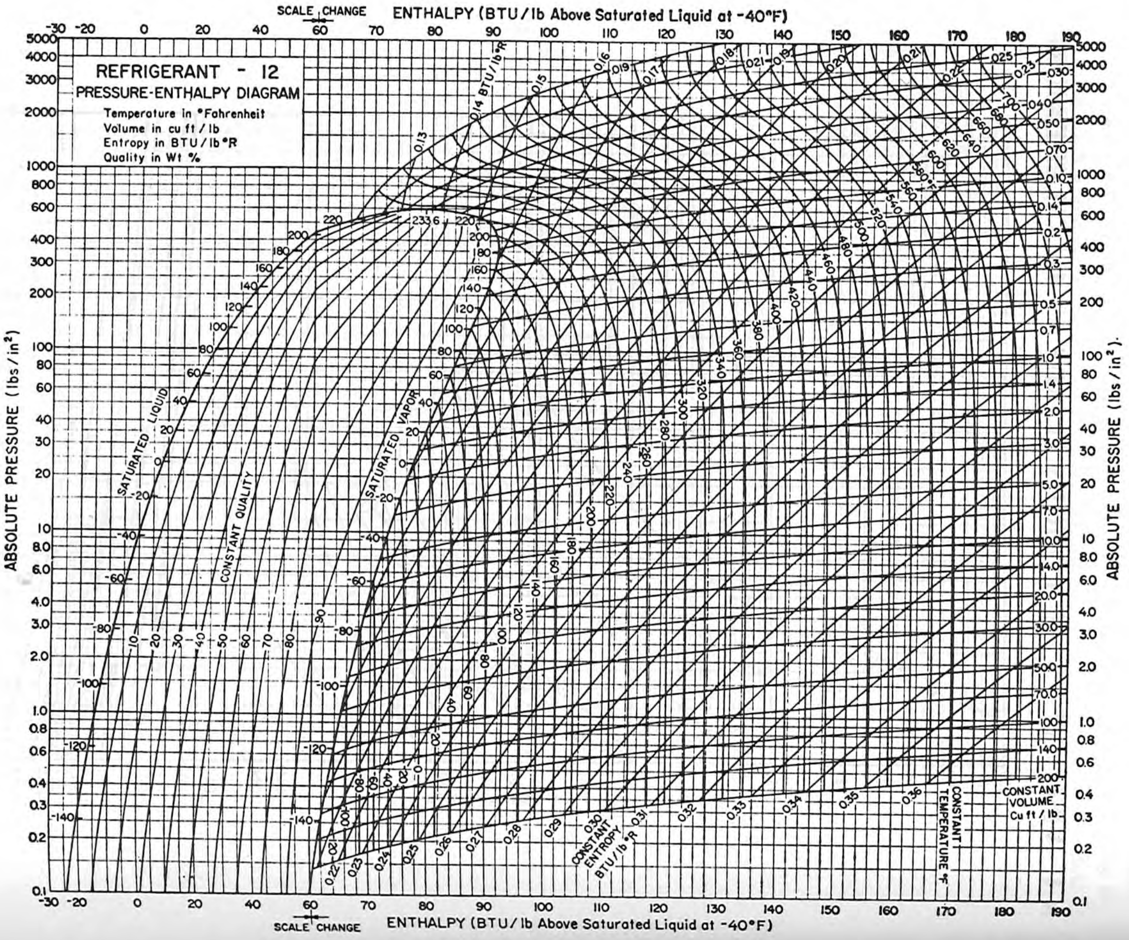


FIGURE 6.21
Economic pipe size for pumped liquids (carbon-steel pipe). Assumptions: Pumping cost = \$135 per horsepower-year; line cost = \$1 per inch of diameter per foot; fixed charges per year on line = 0.40 times line cost; liquid specific gravity = 0.80 (not very critical). Examples: for 150 gal/min and 200 cSt, use the 4-in line; for 150 gal/min and 10 cSt, use the 3-in line. (These prices are from the early 1960s. Current prices are higher, but they have risen more or less together, so that the economic pipe sizes have not changed much.) (Courtesy of the Board of Engineers, Standard Oil Company of California.)

A.2 PRESSURE-ENTHALPY DIAGRAM FOR FREON-12 REFRIGERANT



In the examples in Chapter 4 this plot is read to 4 significant figures. The student is not expected to do this without the tables which accompany this figure, which can be obtained from du Pont.

Courtesy of E. I. du Pont de Nemours and Company, Inc.

A.3 STEEL PIPE DIMENSIONS: CAPACITIES AND WEIGHTS

Nominal pipe size, in	Outside diam., in	Schedule no.	Wall thickness, in	Inside diam., in	Cross-sectional area metal, in²	Inside sectional area, ft²	Circumference, ft, or surface, ft²/ft, of length		Capacity at 1 ft/s velocity		Weight of pipe, lb/ft
							Outside	Inside	U.S. gal/min	lb/h water	
⅛	0.405	40	0.068	0.269	0.072	0.00040	0.106	0.0705	0.179	89.5	0.25
		80	0.095	0.215	0.093	0.00025	0.106	0.0563	0.112	56.0	0.32
¼	0.540	40	0.088	0.364	0.125	0.00072	0.141	0.0954	0.323	161.5	0.43
		80	0.119	0.302	0.157	0.00050	0.141	0.0792	0.224	112.0	0.54
⅜	0.675	40	0.091	0.493	0.167	0.00133	0.177	0.1293	0.596	298.0	0.57
		80	0.126	0.423	0.217	0.00098	0.177	0.1110	0.440	220.0	0.74
½	0.840	40	0.109	0.622	0.250	0.00211	0.220	0.1630	0.945	472.5	0.85
		80	0.147	0.546	0.320	0.00163	0.220	0.1430	0.730	365.0	1.09
		160	0.187	0.466	0.384	0.00118	0.220	0.1220	0.529	264.5	1.31
¾	1.050	40	0.113	0.824	0.333	0.00371	0.275	0.2158	1.665	832.5	1.13
		80	0.154	0.742	0.433	0.00300	0.275	0.1942	1.345	672.5	1.48
		160	0.218	0.614	0.570	0.00206	0.275	0.1610	0.924	462.0	1.94
1	1.315	40	0.133	1.049	0.494	0.00600	0.344	0.2745	2.690	1,345.0	1.68
		80	0.179	0.957	0.639	0.00499	0.344	0.2505	2.240	1,120.0	2.17
		160	0.250	0.815	0.837	0.00362	0.344	0.2135	1.625	812.5	2.85
1¼	1.660	40	0.140	1.380	0.669	0.01040	0.435	0.362	4.57	2,285.0	2.28
		80	0.191	1.278	0.881	0.00891	0.435	0.335	3.99	1,995.0	3.00
		160	0.250	1.160	1.107	0.00734	0.435	0.304	3.29	1,645.0	3.77
1½	1.990	40	0.145	1.610	0.799	0.01414	0.498	0.422	6.34	3,170.0	2.72
		80	0.200	1.500	1.068	0.01225	0.498	0.393	5.49	2,745.0	3.64
		160	0.281	1.338	1.429	0.00976	0.498	0.350	4.38	2,190.0	4.86

(continued)

A.3 (*continued*)

Nominal pipe size, in	Outside diam., in	Schedule no.	Wall thickness, in	Inside diam., in	Cross-sectional area metal, in^2	Inside sectional area, ft^2	Circumference, ft, or surface, ft^2/ft, of length		Capacity at 1 ft/s velocity		Weight of pipe, lb/ft
							Outside	Inside	U.S. gal/min	lb/h water	
2	2.375	40	0.154	2.067	1.075	0.02330	0.622	0.542	10.45	5,225.0	3.66
		80	0.218	1.939	1.477	0.02050	0.622	0.508	9.20	4,600.0	5.03
		160	0.343	1.689	2.190	0.01556	0.622	0.442	6.97	3,485.0	7.45
2½	2.875	40	0.203	2.469	1.704	0.3322	0.753	0.647	14.92	7,460.0	5.80
		80	0.276	2.323	2.254	0.02942	0.753	0.609	13.20	6,600.0	7.67
		160	0.375	2.125	2.945	0.02463	0.753	0.557	11.07	5,535.0	10.0
3	3.500	40	0.216	3.068	2.228	0.05130	0.917	0.804	23.00	11,500.0	7.58
		80	0.300	2.900	3.016	0.04587	0.917	0.760	20.55	10,275.0	10.3
		160	0.437	2.626	4.205	0.03761	0.917	0.688	16.90	8,450.0	14.3
3½	4.000	40	0.226	3.548	2.680	0.06870	1.047	0.930	30.80	15,400.0	9.11
		80	0.318	3.364	3.678	0.06170	1.047	0.882	27.70	13,850.0	12.5
4	4.500	40	0.237	4.026	3.173	0.08840	1.178	1.055	39.6	19,800.0	10.8
		80	0.337	3.826	4.407	0.07986	1.178	1.002	35.8	17,900.0	15.0
		120	0.437	3.626	5.578	0.07170	1.178	0.950	32.2	16,100.0	19.0
		160	0.531	3.438	6.621	0.06447	1.178	0.901	28.9	14,450.0	22.6
5	5.563	40	0.258	5.047	4.304	0.1390	1.456	1.322	62.3	31,150.0	14.7
		80	0.375	4.813	6.112	0.1263	1.456	1.263	57.7	28,850.0	20.8
		120	0.500	4.563	7.953	0.1136	1.456	1.197	51.0	25,500.0	27.1
		160	0.625	4.313	9.696	0.1015	1.456	1.132	45.5	22,750.0	33.0
6	6.625	40	0.280	6.065	5.584	0.2006	1.734	1.590	90.0	45,000.0	19.0
		80	0.432	5.761	8.405	0.1810	1.734	1.510	81.1	40,500.0	28.6
		120	0.562	5.501	10.71	0.1650	1.734	1.445	73.9	36,950.0	36.4
		160	0.718	5.189	13.32	0.1469	1.734	1.360	65.8	32,900.0	45.3
8	8.625	20	0.250	8.125	6.570	0.3601	2.258	2.130	161.5	80,750.0	22.4
		30	0.277	8.071	7.260	0.3553	2.258	2.115	159.4	79,700.0	24.7
		40	0.322	7.981	8.396	0.3474	2.258	2.090	155.7	77,850.0	28.6
		60	0.406	7.813	10.48	0.3329	2.258	2.050	149.4	74,700.0	35.7
		80	0.500	7.625	12.76	0.3171	2.258	2.000	142.3	71,150.0	43.4
		100	0.593	7.439	14.96	0.3018	2.258	1.947	135.3	67,650.0	50.9
		120	0.718	7.189	17.84	0.2819	2.258	1.883	126.5	63,250.0	60.7
		140	0.812	7.001	19.93	0.2673	2.258	1.835	120.0	60,000.0	67.8
		160	0.906	6.813	21.97	0.2532	2.258	1.787	113.5	56,750.0	74.7
10	10.75	20	0.250	10.250	8.24	0.5731	2.814	2.685	257.0	128,500.0	28.1
		30	0.307	10.136	10.07	0.5603	2.814	2.655	252.0	126,000.0	34.3
		40	0.365	10.020	11.90	0.5475	2.814	2.620	246.0	123,000.0	40.5
		60	0.500	9.750	16.10	0.5185	2.814	2.550	233.0	116,500.0	54.8
		80	0.593	9.564	18.92	0.4989	2.814	2.503	224.0	112,000.0	64.4
		100	0.718	9.314	22.63	0.4732	2.814	2.440	212.0	106,000.0	77.0
		120	0.843	9.064	26.24	0.4481	2.814	2.373	201.0	100,500.0	89.2
		140	1.000	8.750	30.63	0.4176	2.814	2.290	188.0	93,750.0	105.0
		160	1.125	8.500	34.02	0.3941	2.814	2.230	177.0	88,500.0	116.0
12	12.75	20	0.250	12.250	9.82	0.8185	3.338	3.31	367.0	183,500.0	33.4
		30	0.330	12.090	12.87	0.7972	3.338	3.17	358.0	179,000.0	43.8
		40	0.406	11.938	15.77	0.7773	3.338	3.13	349.0	174,500.0	53.6
		60	0.562	11.626	21.52	0.7372	3.338	3.05	331.0	165,500.0	73.2
		80	0.687	11.376	26.03	0.7058	3.338	2.98	317.0	158,500.0	88.6
		100	0.843	11.064	31.53	0.6677	3.338	2.90	299.0	149,500.0	108.0
		120	1.000	10.750	36.91	0.6303	3.338	2.82	283.0	141,500.0	126.0
		140	1.125	10.500	41.08	0.6013	3.338	2.75	270.0	135,000.0	140.0
		160	1.312	10.126	47.14	0.5592	3.338	2.66	251.0	125,500.0	161.0
14	14.0	10	0.250	13.500	10.80	0.9940	3.665	3.54	446.0	223,000.0	36.8
		20	0.312	13.376	13.42	0.9750	3.665	3.51	438.0	219,000.0	45.7
		30	0.375	13.250	16.05	0.9575	3.665	3.47	430.0	215,000.0	54.6
		40	0.437	13.126	18.61	0.9397	3.665	3.44	422.0	211,000.0	63.3
		60	0.593	12.814	24.98	0.8956	3.665	3.36	402.0	201,000.0	85.0
		80	0.750	12.500	31.22	0.8522	3.665	3.28	382.0	191,000.0	107.0
		100	0.937	12.126	38.45	0.8020	3.665	3.18	360.0	180,000.0	131.0
		120	1.062	11.876	43.17	0.7693	3.665	3.11	345.0	172,500.0	147.0
		140	1.250	11.500	50.07	0.7213	3.665	3.01	324.0	162,000.0	171.0
		160	1.406	11.188	55.63	0.6827	3.665	2.93	306.0	153,000.0	190.0
16	16.0	10	0.250	15.500	12.37	1.3104	4.189	4.06	587.0	293,500.0	42.1
		20	0.312	15.376	15.38	1.2895	4.189	4.03	578.0	289,000.0	52.3
		30	0.375	15.250	18.41	1.2680	4.189	4.00	568.0	284,000.0	62.6
		40	0.500	15.000	24.35	1.2272	4.189	3.93	550.0	275,000.0	82.8

(*continued*)

A.3 *(continued)*

Nominal pipe size, in	Outside diam., in	Schedule no.	Wall thickness, in	Inside diam., in	Cross-sectional area metal, in^2	Inside sectional area, ft^2	Circumference, ft, or surface, ft^2/ft, of length		Capacity at 1 ft/s velocity		Weight of pipe, lb/ft
							Outside	Inside	U.S. gal/min	lb/h water	
		60	0.656	14.688	31.62	1.1766	4.189	3.85	528.0	264,000.0	108.0
		80	0.843	14.314	40.14	1.1175	4.189	3.76	500.0	250,000.0	137.0
		100	1.031	13.938	48.48	1.0596	4.189	3.65	474.0	237,000.0	165.0
		120	1.218	13.564	56.56	1.0035	4.189	3.56	450.0	225,000.0	193.0
		140	1.437	13.126	65.74	0.9397	4.189	3.44	422.0	211,000.0	224.0
		160	1.562	12.876	70.85	0.9043	4.189	3.37	405.0	202,500.0	241.0
18	18.0	10	0.250	17.50	13.94	1.6703	4.712	4.59	748.0	374,000.0	47.4
		20	0.312	17.376	17.34	1.6468	4.712	4.55	738.0	369,000.0	59.0
		30	0.437	17.126	24.11	1.5993	4.712	4.49	717.0	358,500.0	82.0
		40	0.562	16.876	30.79	1.5533	4.712	4.42	697.0	348,500.0	105.0
		60	0.718	15.564	38.98	1.4964	4.712	4.34	670.0	335,000.0	133.0
		80	0.937	16.126	50.23	1.4183	4.712	4.23	635.0	317,500.0	171.0
		100	1.156	15.688	61.17	1.3423	4.712	4.11	602.0	301,000.0	208.0
		120	1.343	15.314	70.28	1.2791	4.712	4.02	573.0	286,500 0	239.0
		140	1.562	14.876	80.66	1.2070	4.712	3.90	540.0	270,000.0	275.0
		160	1.750	14.500	89.34	1.1467	4.712	3.80	514.0	257,000.0	304.0
20	20.0	10	0.250	19.500	15.51	2.0740	5.236	5.11	930.0	465,000.0	52.8
		20	0.375	19.250	23.12	2.0211	5.236	5.05	902.0	451,000.0	78.6
		30	0.500	19.000	30.63	1.9689	5.236	4.98	883.0	441,500.0	105.0
		40	0.593	18.814	36.15	1.9305	5.236	4.94	866.0	433,000.0	123.0
		60	0.812	18.376	48.95	1.8317	5.236	4.81	826.0	413,000.0	167.0
		80	1.031	17.938	61.44	1.7550	5.236	4.70	787.0	393,500.0	209.0
		100	1.250	17.500	73.63	1.6703	5.236	4.59	750.0	375,000.0	251.0
		120	1.500	17.000	87.18	1.5762	5.236	4.46	707.0	353,500.0	297.0
		140	1.750	16.500	100.3	1.4849	5.236	4.32	665.0	332,500.0	342.0
		160	1.937	16.126	109.9	1.4183	5.236	4.22	635.0	317,500.0	374.0

The schedule number corresponds roughly to 10^3 allowable pressure per allowable stress. Thus, for a material with an allowable stress of 10,000 lb/in^2 a schedule 40 pipe would have an allowable pressure of 400 lb/in^2. [From *Chemical Engineers' Handbook*, by Perry, Chilton, and Kirkpatrick. Copyright © 1963, McGraw-Hill, Inc. Used by permission of the publisher.]

A.4 FLOW OF WATER THROUGH SCHEDULE 40 STEEL PIPE

Discharge		Pressure drop per 100 ft and velocity in schedule 40 pipe for water at 60°F															
gal/min	ft^3/s	Velocity, ft/s	Press. drop, lb/in^2	Velocity, ft/s	Press. drop, lb/in^2	Velocity, ft/s	Press. drop, lb/in^2	Velocity, ft/s	Press. drop, lb/in^2	Velocity, ft/s	Press. drop, lb/in^2	Velocity, ft/s	Press. drop, lb/in^2	Velocity, ft/s	Press. drop, lb/in^2	Velocity, ft/s	Press. drop, lb/in^2
		$\frac{1}{8}$ in		$\frac{1}{4}$ in													
0.2	0.000446	1.13	1.86	0.616	0.359	$\frac{3}{8}$ in		$\frac{1}{2}$ in									
0.3	0.000668	1.69	4.22	0.924	0.903	0.504	0.159	0.317	0.061								
0.4	0.000891	2.26	6.98	1.23	1.61	0.672	0.345	0.422	0.086	$\frac{3}{4}$ in							
0.5	0.00111	2.82	10.5	1.54	2.39	0.840	0.539	0.528	0.167	0.301	0.033						
0.6	0.00134	3.39	14.7	1.85	3.29	1.01	0.751	0.633	0.240	0.361	0.041						
0.8	0.00178	4.52	25.0	2.46	5.44	1.34	1.25	0.844	0.408	0.481	0.102	1 in		$1\frac{1}{4}$ in		$1\frac{1}{2}$ in	
1	0.00223	5.65	37.2	3.08	8.28	1.68	1.85	1.06	0.600	0.602	0.155	0.371	0.048	0.429	0.044		
2	0.00446	11.29	134.4	6.16	30.1	3.36	6.58	2.11	2.10	1.20	0.526	0.743	0.164	0.644	0.090		
3	0.00668			9.25	64.1	5.04	13.9	3.17	4.33	1.81	1.09	1.114	0.336	0.858	0.150	0.473	0.043
4	0.00891	2 in		12.33	111.2	6.72	23.9	4.22	7.42	2.41	1.83	1.49	0.565	1.073	0.223	0.630	0.071
5	0.01114					8.40	36.7	5.28	11.2	3.01	2.75	1.86	0.835			0.788	0.104
6	0.01337	0.574	0.044	$2\frac{1}{2}$ in		10.08	51.9	6.33	15.8	3.61	3.84	2.23	1.17	1.29	0.309	0.946	0.145
8	0.01782	0.765	0.073			13.44	91.1	8.45	27.7	4.81	6.60	2.97	1.99	1.72	0.518	1.26	0.241
10	0.02228	0.956	0.108	0.670	0.046			10.56	42.4	6.02	9.99	3.71	2.99	2.15	0.774	1.58	0.361
15	0.03342	1.43	0.224	1.01	0.094	3 in				9.03	21.6	5.57	6.36	3.22	1.63	2.37	0.755
20	0.04456	1.91	0.375	1.34	0.158	0.868	0.056	$3\frac{1}{2}$ in		12.03	37.8	7.43	10.9	4.29	2.78	3.16	1.28
25	0.05570	2.39	0.561	1.68	0.234	1.09	0.083	0.812	0.041	4 in		9.28	16.7	5.37	4.22	3.94	1.93
30	0.06684	2.87	0.786	2.01	0.327	1.30	0.114	0.974	0.056			11.14	23.8	6.44	5.92	4.73	2.72
35	0.07798	3.35	1.05	2.35	0.436	1.52	0.151	1.14	0.704	0.882	0.041	12.99	32.2	7.51	7.90	5.52	3.64
40	0.08912	3.83	1.35	2.68	0.556	1.74	0.192	1.30	0.095	1.01	0.052	14.85	41.5	8.59	10.24	6.30	4.65
45	0.1003	4.30	1.67	3.02	0.668	1.95	0.239	1.46	0.117	1.13	0.064			9.67	12.80	7.09	5.85
50	0.1114	4.78	2.03	3.35	0.839	2.17	0.288	1.62	0.142	1.26	0.076	5 in		10.74	15.66	7.88	7.15
60	0.1337	5.74	2.87	4.02	1.18	2.60	0.406	1.95	0.204	1.51	0.107			12.89	22.2	9.47	10.21
70	0.1560	6.70	3.84	4.69	1.59	3.04	0.540	2.27	0.261	1.76	0.143	1.12	0.047			11.05	13.71
80	0.1782	7.65	4.97	5.36	2.03	3.47	0.687	2.60	0.334	2.02	0.180	1.28	0.060			12.62	17.59
90	0.2005	8.60	6.20	6.03	2.53	3.91	0.861	2.92	0.416	2.27	0.224	1.44	0.074	6 in		14.20	22.0
100	0.2228	9.56	7.59	6.70	3.09	4.34	1.05	3.25	0.509	2.52	0.272	1.60	0.099	1.11	0.036	15.78	26.9
125	0.2785	11.97	11.76	8.38	4.71	5.43	1.61	4.06	0.769	3.15	0.415	2.01	0.135	1.39	0.055	19.72	41.4
150	0.3342	14.36	16.70	10.05	6.69	6.51	2.24	4.87	1.08	3.78	0.580	2.41	0.190	1.67	0.077		
175	0.3899	16.75	22.3	11.73	8.97	7.60	3.00	5.68	1.44	4.41	0.774	2.81	0.253	1.94	0.102		
200	0.4456	19.14	28.8	13.42	11.68	8.68	3.87	6.49	1.85	5.04	0.985	3.21	0.323	2.22	0.130	8 in	
225	0.5013	...	...	15.09	14.63	9.77	4.83	7.30	2.32	5.67	1.23	3.61	0.401	2.50	0.162	1.44	0.043
250	0.557	...	...	...	...	10.85	5.93	8.12	2.84	6.30	1.46	4.01	0.495	2.78	0.195	1.60	0.051
275	0.6127	...	...	...	...	11.94	7.14	8.93	3.40	6.93	1.79	4.41	0.583	3.05	0.234	1.76	0.061
300	0.6684	...	...	...	...	13.00	8.36	9.74	4.02	7.56	2.11	4.81	0.683	3.33	0.275	1.92	0.072
325	0.7241	...	...	...	...	14.12	9.89	10.53	4.09	8.19	2.47	5.21	0.797	3.61	0.320	2.08	0.083

(*continued*)

A.4 (*continued*)

Discharge		Pressure drop per 100 ft and velocity in schedule 40 pipe for water at 60°F															
gal/min	ft³/s	Velocity, ft/s	Press. drop, lb/in²	Velocity, ft/s	Press. drop, lb/in²	Velocity, ft/s	Press. drop, lb/in²	Velocity, ft/s	Press. drop, lb/in²	Velocity, ft/s	Press. drop, lb/in²	Velocity, ft/s	Press. drop, lb/in²	Velocity, ft/s	Press. drop, lb/in²	Velocity, ft/s	Press. drop, lb/in²
350	0.7798			...	...	...	...	11.36	5.41	8.82	2.84	5.62	0.919	3.89	0.367	2.24	0.095
375	0.8355			...	...	...	...	12.17	6.18	9.45	3.25	6.02	1.05	4.16	0.416	2.40	0.108
400	0.8912			...	...	...	...	12.98	7.03	10.08	3.68	6.42	1.19	4.44	0.471	2.56	0.121
425	0.9469			...	...	...	...	13.80	7.89	10.71	4.12	6.82	1.33	4.72	0.529	2.73	0.136
450	1.003	10 in		...	...	...	...	14.61	8.80	11.34	4.60	7.22	1.48	5.00	0.590	2.89	0.151
475	1.059	1.93	0.054			...	...	...	...	11.97	5.12	7.62	1.64	5.27	0.653	3.04	0.166
500	1.114	2.03	0.059			...	...	...	...	12.60	5.65	8.02	1.81	5.55	0.720	3.21	0.182
550	1.225	2.24	0.071			...	...	...	...	13.85	6.79	8.82	2.17	6.11	0.861	3.53	0.219
600	1.337	2.44	0.083			...	...	...	...	15.12	8.04	9.63	2.55	6.66	1.02	3.85	0.258
650	1.448	2.64	0.097	12 in		...	...	...	...	...	...	10.43	2.98	7.22	1.18	4.17	0.301
700	1.560	2.85	0.112	2.01	0.047			...	...	...	...	11.23	3.43	7.78	1.35	4.49	0.343
750	1.671	3.05	0.127	2.15	0.054	14 in		...	...	...	...	12.03	3.92	8.33	1.55	4.81	0.392
800	1.782	3.25	0.143	2.29	0.061			...	...	...	...	12.83	4.43	8.88	1.75	5.13	0.443
850	1.894	3.46	0.160	2.44	0.068	2.02	0.042	...	...	...	...	13.64	5.00	9.44	1.96	5.45	0.497
900	2.005	3.66	0.179	2.58	0.075	2.13	0.047	...	...	...	...	14.44	5.58	9.99	2.18	5.77	0.554
950	2.117	3.86	0.198	2.72	0.083	2.25	0.052			...	...	15.24	6.21	10.55	2.42	6.09	0.613
1,000	2.228	4.07	0.218	2.87	0.091	2.37	0.057	16 in		...	...	16.04	6.84	11.10	2.68	6.41	0.675
1,100	2.451	4.48	0.260	3.15	0.110	2.61	0.068			...	...	17.65	8.23	12.22	3.22	7.05	0.807
1,200	2.674	4.88	0.306	3.44	0.128	2.85	0.080	2.18	0.042	...	...	...	...	13.33	3.81	7.70	0.948
1,300	2.896	5.29	0.355	3.73	0.150	3.08	0.093	2.36	0.048	...	...	...	...	14.43	4.45	8.33	1.11
1,400	3.119	5.70	0.409	4.01	0.171	3.32	0.107	2.54	0.055					15.55	5.13	8.98	1.28
1,500	3.342	6.10	0.466	4.30	0.195	3.56	0.122	2.72	0.063	18 in				16.66	5.85	9.62	1.46
1,600	3.565	6.51	0.527	4.59	0.219	3.79	0.138	2.90	0.071					17.77	6.61	10.26	1.65
1,800	4.010	7.32	0.663	5.16	0.276	4.27	0.172	3.27	0.088	2.58	0.050			19.99	8.37	11.54	2.08
2,000	4.456	8.14	0.808	5.73	0.339	4.74	0.209	3.63	0.107	2.87	0.060	20 in		22.21	10.3	12.82	2.55
2,500	5.570	10.17	1.24	7.17	0.515	5.93	0.321	4.54	0.163	3.59	0.091					16.03	3.94
3,000	6.684	12.20	1.76	8.60	0.731	7.11	0.451	5.45	0.232	4.30	0.129	3.46	0.075	24 in		19.24	5.59
3,500	7.798	14.24	2.38	10.03	0.982	8.30	0.607	6.35	0.312	5.02	0.173	4.04	0.101			22.44	7.56
4,000	8.912	16.27	3.08	11.47	1.27	9.48	0.787	7.26	0.401	5.74	0.222	4.62	0.129	3.19	0.052	25.65	9.80
4,500	10.03	18.31	3.87	12.90	1.60	10.67	0.990	8.17	0.503	6.46	0.280	5.20	0.162	3.59	0.065	28.87	12.2
5,000	11.14	20.35	4.71	14.33	1.95	11.85	1.21	9.08	0.617	7.17	0.340	5.77	0.199	3.99	0.079	...	...
6,000	13.37	24.41	6.74	17.20	2.77	14.23	1.71	10.89	0.877	8.61	0.483	6.93	0.280	4.79	0.111	...	...
7,000	15.60	28.49	9.11	20.07	3.74	16.60	2.31	12.71	1.18	10.04	0.652	8.08	0.376	5.59	0.150	...	...
8,000	17.82	...	...	22.93	4.84	18.96	2.99	14.52	1.51	11.47	0.839	9.23	0.488	6.38	0.192	...	...
9,000	20.05	...	...	25.79	6.09	21.34	3.76	16.34	1.90	12.91	1.05	10.39	0.608	7.18	0.242	...	...
10,000	22.28	...	...	28.66	7.46	23.71	4.61	18.15	2.34	14.34	1.28	11.54	0.739	7.98	0.294	...	...
12,000	26.74	...	...	34.40	10.7	28.45	6.59	21.79	3.33	17.21	1.83	13.85	1.06	9.58	0.416	...	...
14,000	31.19	...	...	...	...	33.19	8.89	25.42	4.49	20.08	2.45	16.16	1.43	11.17	0.562	...	...
16,000	35.65	...	...	...	...	...	...	29.05	5.83	22.95	3.18	18.47	1.85	12.77	0.723	...	...
18,000	40.10	...	...	...	...	...	...	32.68	7.31	25.82	4.03	20.77	2.32	14.36	0.907	...	...
20,000	44.56	...	...	...	...	...	...	36.31	9.03	28.69	4.93	23.08	2.86	15.96	1.12	...	...

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A.5 COMPRESSIBLE-FLOW TABLES FOR $k = 1.4$

$\mathcal{M}$	$\frac{P}{P_R}$	$\frac{\rho}{\rho_R}$	$\frac{T}{T_R}$	$\frac{A}{A^*}$	$\frac{V}{c^*}$	$\mathcal{M}$	$\frac{P}{P_R}$	$\frac{\rho}{\rho_R}$	$\frac{T}{T_R}$	$\frac{A}{A^*}$	$\frac{V}{c^*}$
0	1.0000	1.0000	1.0000	∞	0	0.25	0.9575	0.9694	0.9877	2.4027	0.27217
0.01	0.9999	1.0000	1.0000	57.8738	0.01095	0.26	0.9541	0.9670	0.9867	2.3173	0.28294
0.02	0.9997	0.9998	0.9999	28.9421	0.02191	0.27	0.9506	0.9645	0.9856	2.2385	0.29361
0.03	0.9994	0.9996	0.9998	19.3005	0.03286	0.28	0.9470	0.9619	0.9846	2.1656	0.30435
0.04	0.9989	0.9992	0.9997	14.4815	0.04381	0.29	0.9433	0.9592	0.9835	2.0979	0.31504
0.05	0.9983	0.9988	0.9995	11.5014	0.05476	0.30	0.9395	0.9564	0.9823	2.0351	0.32572
0.06	0.9975	0.9982	0.9993	9.6659	0.06570	0.31	0.9355	0.9535	0.9811	1.9765	0.33637
0.07	0.9966	0.9976	0.9990	8.2915	0.97664	0.32	0.9315	0.9506	0.9799	1.9219	0.34701
0.08	0.9955	0.9968	0.9987	7.2616	0.08758	0.33	0.9274	0.9476	0.9787	1.8707	0.35762
0.09	0.9944	0.9960	0.9984	6.4613	0.09851	0.34	0.9231	0.9445	0.9774	1.8229	0.36822
0.10	0.9930	0.9950	0.9980	5.8218	0.10944	0.35	0.9188	0.9413	0.9761	1.7780	0.37879
0.11	0.9916	0.9940	0.9976	5.2992	0.12035	0.36	0.9143	0.9380	0.9747	1.7358	0.38935
0.12	0.9900	0.9928	0.9971	4.8643	0.13126	0.37	0.9098	0.9347	0.9733	1.6961	0.39988
0.13	0.9883	0.9916	0.9966	4.4969	0.14217	0.38	0.9052	0.9313	0.9719	1.6587	0.41039
0.14	0.9864	0.9903	0.9961	4.1824	0.15306	0.39	0.9004	0.9278	0.9705	1.6234	0.42087
0.15	0.9844	0.9888	0.9955	3.9103	0.16395	0.40	0.8956	0.9243	0.9690	1.5901	0.43133
0.16	0.9823	0.9873	0.9949	3.6727	0.17482	0.41	0.8907	0.9207	0.9675	1.5587	0.44177
0.17	0.9800	0.9857	0.9943	3.4635	0.18569	0.42	0.8857	0.9170	0.9659	1.5289	0.45218
0.18	0.9776	0.9840	0.9936	3.2779	0.19654	0.43	0.8807	0.9132	0.9643	1.5007	0.46257
0.19	0.9751	0.9822	0.9928	3.1123	0.20739	0.44	0.8755	0.9094	0.9627	1.4740	0.47293
0.20	0.9725	0.9803	0.9921	2.9635	0.21822	0.45	0.8703	0.9055	0.9611	1.4487	0.48326
0.21	0.9697	0.9783	0.9913	2.8293	0.22904	0.46	0.8650	0.9016	0.9594	1.4246	0.49357
0.22	0.9668	0.9762	0.9904	2.7076	0.23984	0.47	0.8596	0.8976	0.9577	1.4048	0.50385
0.23	0.9638	0.9740	0.9895	2.5968	0.25063	0.48	0.8541	0.8935	0.9560	1.3801	0.51410
0.24	0.9607	0.9718	0.9886	2.4956	0.26141	0.49	0.8486	0.8894	0.9542	1.3595	0.52433

(continued)