ChE 374–Lecture 17–Turbulent Pipe Flows

- Recap
  - Pipes → 2 Π's: Re, f: $f = f(Re)$.
  - Darcy friction factor: $f = \frac{2\Delta P_L D}{L\rho v^2}$.
  - For pipes: $\frac{\Delta P}{\rho} + g\Delta z = -\frac{fLv^2}{2D}$.
    * Follows from the definition of f, and the friction/loss term in the mechanical energy equation. Can add on the work terms too.
- Turbulent Pipe flow
  - High Re, Chaotic, Unsteady, Random velocity, velocity fluctuations, 3D
  - Unsteady, and 3D → steady and 1D when take average
    * $u = \bar{u} + u\prime$.
  - Turbulent profile is flatter in the center and steeper near the walls than laminar.
    * Higher τ_w, higher friction, higher pressure drop.
    * Recall: laminar transports by diffusion (slow), turbulence transports by convection (fast): Re is ratio of convection rate to diffusion rate, and Re is high.
- Velocity Profile
  - Four regions: (1) inner/viscous sublayer; (2) buffer layer; (3) overlap/log-law; (4) Outer/turbulent.
  - Scale u and y using viscous scales: $u_\tau = \sqrt{\tau_w/\rho}$, $\delta = \nu/u_\tau$.
  - Dimensional analysis tells us the shape of profiles in regions 1, 3.
- Shear Stress
  - Need to account for fluctuations, which move the velocity, hence momentum (τ is a momentum flux, recall).
  - $\tau_t = -\rho\overline{u\prime v\prime}$.
    * Units of stress (F/A)
    * Called a Reynolds Stress
    * Flux of u-momentum, due to v-fluctuations (u is streamwise, v toward wall).
  - Model with $\tau = \mu_t \frac{d\bar{u}}{dy}$
    * μ_t is the eddy viscosity, which is a Flow property, NOT a fluid property.
- Pipe Roughness
  - The very thin region of viscous effects in turbulent flow means pipe wall roughness is important.
  - Irregularities in the surface disturb the viscous layer → more friction.
  - ϵ is the roughness and ϵ/D is a new parameter.
  - $f = f(Re, \epsilon/D)$.
- Colbrook Equation / Moody Chart
  - Colbrook equation: $\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\epsilon/D}{3/7} + \frac{2.51}{Re\sqrt{f}}\right)$.
    * Implicit in f; use solver, etc. to solve for f.
  - Haaland Equation: $\frac{1}{\sqrt{f}} = -1.8\log\left(\frac{6.9}{Re} + \left(\frac{\epsilon/D}{3.7}\right)^1 .11\right)$.
    * Explicit in f, and accurate to within 2%.
  - Moody Diagram
    * Be familiar with properties: laminar, transition, f(Re) dependence, f(ϵ) dependence, limiting cases of $\epsilon = 0$ and high Re.

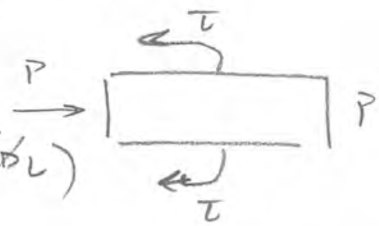
Class 17 - Turbulent Pipe Flow

Outline

- Recap
- Turbulent Flow
  - Fluctuations
  - Velocity profile
  - Shear stress
- friction Factor

- Recap: Dimensional Analysis of Pipe flow
  - $\frac{\Delta P}{L}$, D, V, ρ, μ $\longrightarrow$ 2 Groups Re, $\frac{\Delta P D}{L \rho V^2}$
  - Derived Laminar Flow eqns

Force Bal:



$$\Delta P \left(\frac{\pi}{4} D^2\right) = \tau_w (\pi D L)$$

$$\rightarrow \frac{\Delta P D}{L} = 4 \tau_w$$

- Group $\frac{\Delta P D}{L \rho V^2} = \frac{4 \tau_w}{\rho V^2}$; $\frac{\rho V^2}{2}$ instead $\rightarrow$ $\frac{8 \tau_w}{\rho V_2} = f$

- Laminar flow $\rightarrow$ $f = \frac{64}{Re}$

Now: Flow $\longrightarrow$ ΔP $\longrightarrow$ Power

Flow $\longrightarrow$ Re $\longrightarrow$ f $\longrightarrow$ ΔP $\longrightarrow$ Power.

$$\frac{\Delta P}{\rho} = \frac{-f L V^2}{2D} \qquad h_L = \frac{-f L V^2}{2 D g}$$

$$\dot{W} = \dot{m} \frac{\Delta P}{\rho} = \dot{m} g h_L$$

Turbulent Pipe Flow

- High Re
- Chaotic
- Unsteady
- Random Vel
- Strong Vel fluctuations
- 3D

- K. H. movie

- Chaotic / random → use mean flow + fluctuation.
  - $u = \bar{u} + u'$

  - Laminar (means) · turbulent

  - turb eddies Flatten profile, but still no slip at wall → Steeper at wall.
  - higher τ_w
  - higher Friction
  - higher ΔP_{loss}

- Slide → eddies smaller near wall
- Movie ODT → Eddies transport momentum → momentum flux.

Laminar → $\tau = -\mu \frac{du}{dy}$

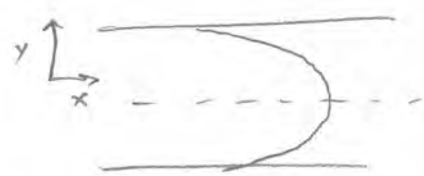
Turbulent → $\tau = -\mu \frac{d\bar{u}}{dy}$? → too low.

- even though $\frac{d\bar{u}}{dy}$ is higher Than in Laminar
- neglects The eddies

→

turbulent eddies transport momentum.

$$\dot{mom} = \dot{m}u' = (\rho v' A)u'$$



$$\rightarrow \tau_t = \rho \overline{v'u'}$$

- units of stress F/A
- "Reynolds stress
- Don't know it → model it

$\tau_t = \mu_t \frac{d\bar{u}}{dy}$ · where μ_t is an eddy viscosity
· μ_t is a property of the flow Not the fluid.

Slide - Avg vs instantaneous.
- Velocity regions of the flow

* Note how Steep Profile near wall

Eddies are small near wall

(Q). what impact will a rough wall have?
Corrosion, Scale build up, etc.

→ roughness interferes with turbulence, not just Viscous sublayer

(Q) → higher or lower friction ?

How to characterize ?

→ new length Scale · ϵ

→ new group ϵ/D

→ $f = f(Re, \epsilon/D)$

Colbrook Equation / Moody Chart.

$f = f(Re, \epsilon/D)$

$$\frac{1}{\sqrt{f}} = -2.0 \log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) \qquad \text{Colbrook Eq.}$$

· Implicit in f → use solver, etc

$$\frac{1}{\sqrt{f}} = -1.8 \log_{10}\left(\frac{6.9}{Re} + \left(\frac{\epsilon/D}{3.7}\right)^{1.11}\right) \qquad \text{Haaland Eqn.}$$

· Explicit in f

Moody Chart.

1. Laminar line
2. Transition region
3. f Decreases with Re
4. f increases with ϵ/D
5. f levels off with Re
6. Limiting cases, $\epsilon = 0$, $Re \to \infty$

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3 → f drops with Re, but friction Does not

$\frac{\Delta P}{\rho} = \frac{fLV^2}{2D}$; $Re \propto V$; as Re ↑, V^2 ↑ faster than f Decreases.

5 → $\delta_{visc} < \epsilon$ → ϵ roughness Dominates effect of viscous sublayer → indep. of Re, which increases turb, but viscosity is at the wall

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1 Eqn, 1 unknown $\quad \frac{\Delta P}{\rho} = \frac{fLV^2}{2D}$

Problem Types

(More on this later)

① Find ΔP given D, L, V
<u>easy</u> : D, V → Re → f → ΔP

② Find V given D, L, ΔP
<u>harder</u> : Guess V → Re, → f → (ΔP) → Repeat

③ Find D given L, ΔP, V
<u>hardest</u> : Guess D → Re → f → (D) → repeat

Chemical Engineering 374

Fluid Mechanics

Turbulent Pipe Flows

Big whorls have little whorls
That feed on their velocity
And little whorls have lesser whorls
And so on to viscosity
--Lewis Richardson

I am an old man now, and when I die and go to heaven there are
two matters on which I hope for enlightenment.
One is quantum electrodynamics, and the other is the
Turbulent motion of fluids.
About the former I am rather optimistic.
--Attr. to Horace Lamb

Turbulence examples

3

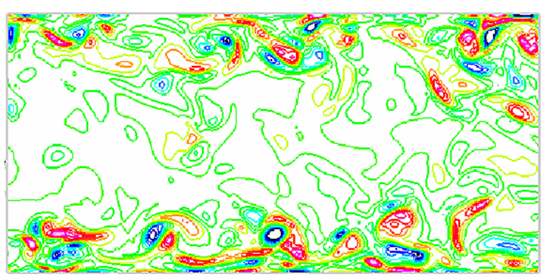
Turbulent Mixing Layer

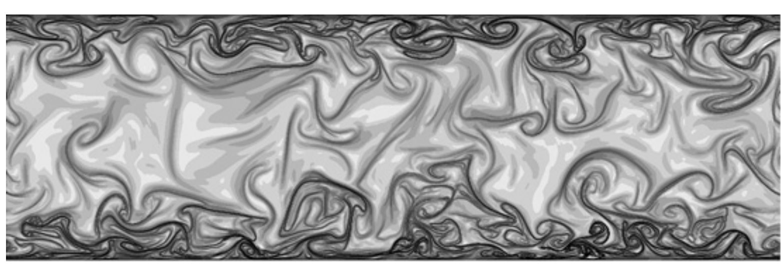
Flow

Flow

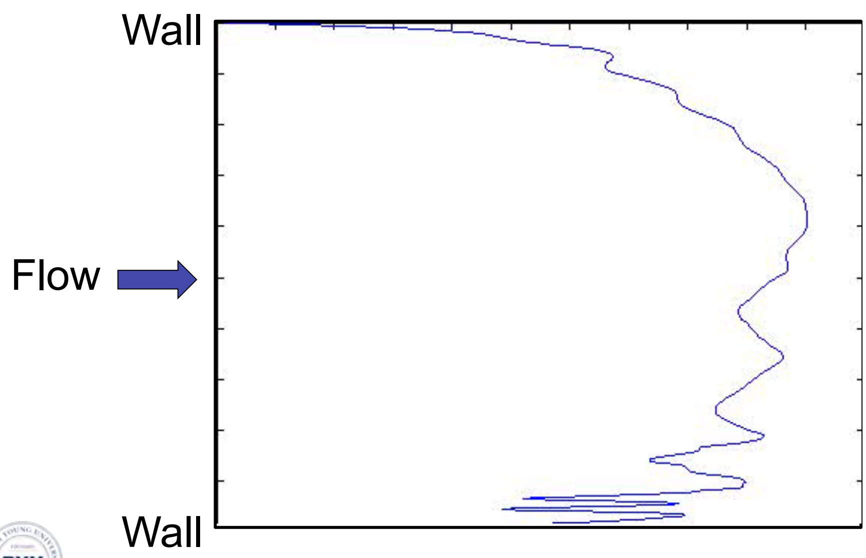
4

Channel Flow Simulations

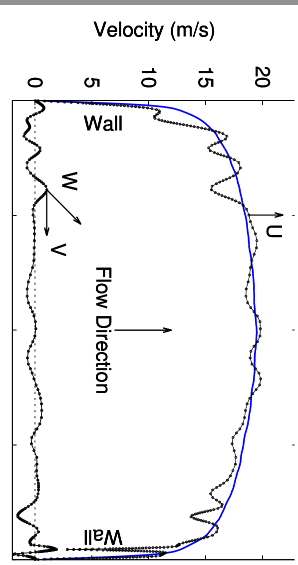


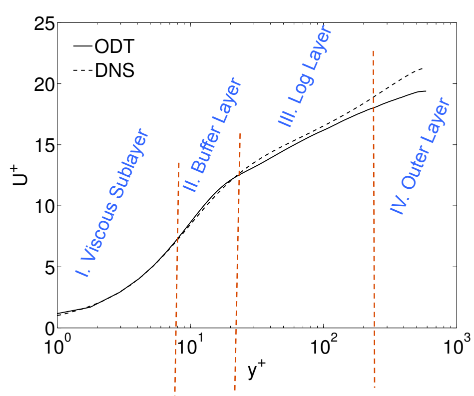


Channel Flow Simulation



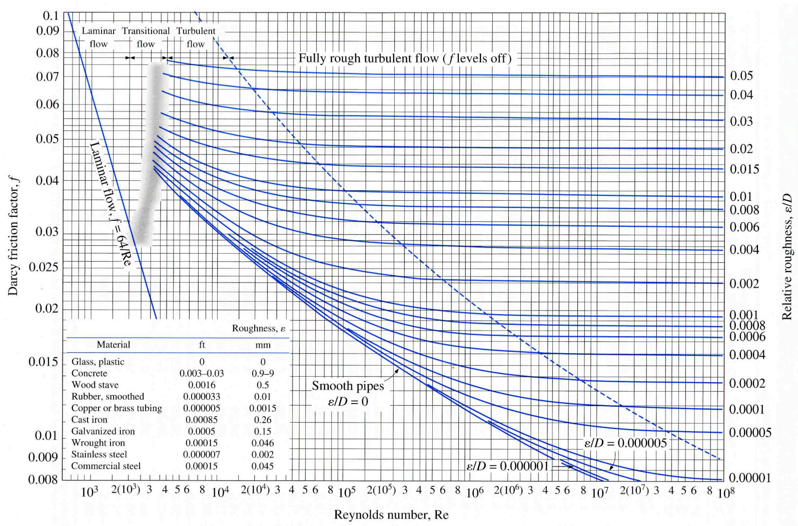
Instantaneous and Mean





Shape of Regions I and III from dimensional analysis!

Moody Chart



Material	Roughness, ε ft	mm
Glass, plastic	0	0
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

FIGURE A–12

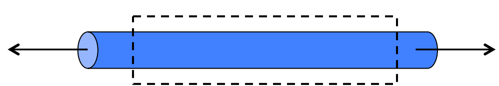
The Moody chart for the friction factor for fully developed flow in circular pipes for use in the head loss relation $h_L = f\frac{L}{D}\frac{V^2}{2g}$. Friction factors in the turbulent flow are evaluated from the Colebrook equation $\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$.

Summary

$$\left(\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2} + g\Delta z\right) = \frac{\dot{W}_u}{\dot{m}} - \frac{\dot{F}}{\dot{m}}$$

$$\left(\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2} + g\Delta z\right) = \frac{\dot{W}_u}{\dot{m}} - \frac{fLv^2}{2D}$$

$$\frac{\Delta P_L}{\rho} = -\frac{fLv^2}{2D}$$

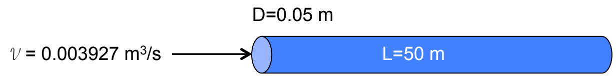


- Consider pipe flow.
- Question: How to simplify the above equation?
- Friction balances pressure drop.
- To find pressure drop for pipe/fluid, and velocity (flow rate):
  - Get f
    - f = f(Re)
    - Get Re = ρDv/μ
  - Plug solve equation for ΔP_L

Colebrook $$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

Haaland $$\frac{1}{\sqrt{f}} = -1.8\log\left[\frac{6.9}{Re} + \left(\frac{\epsilon/D}{3.7}\right)^{1.11}\right]$$

Example



- Water flow in a smooth pipe:
- ρ = 1000 kg/m^3,
- μ = 0.001 kg/m*s → ν=1E-6 m^2/s

- Find ΔP_L, h_L, Power

- Question: What is happening physically?
- Question: What do I know?
- Question: What do I want?
- Question: What relationships do I have?

- Solve Re → f →ΔP_L → h_L→ Power
- Re = ρDv/μ:
  - Need v:
  - v = $\mathcal{V}$ / A = $\mathcal{V}$/ (πD^2/4) = 2 m/s
  - Re = 100,000
- Colbrook → f = 0.0178
- ΔP = fLρv^2/2D = 35600 Pa
- h_L =ΔP/ρg = 3.63 m
- Power = $\mathcal{V}\Delta$P = 140 W

Colebrook $$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

Haaland $$\frac{1}{\sqrt{f}} = -1.8\log\left[\frac{6.9}{Re} + \left(\frac{\epsilon/D}{3.7}\right)^{1.11}\right]$$

$$\frac{\Delta P_L}{\rho} = -\frac{fLv^2}{2D}$$