ChE 374–Lecture 16–Laminar Flow

- Outline
  - Chapter 8 takes us through the next exam
  - Laminar Pipe Flows (today)
  - Turbulent Pipe Flows
  - Minor Losses (valves, fittings, etc.)
  - Pipe Networks
  - Flow Meters.
- Reynolds Number ($Re = \rho Dv/\mu$, or $Re = Dv/\nu$). YOU MUST KNOW THIS.
  - Characterizes pipe flow
    - Turbulence transition at Re=2300 in pipe flows.
    - Ratio of inertia to viscous forces, or ratio of diffusive to convective timescales.
- Entrance Region
  - Flows must develop: Steady, but changing downstream until fully developed.
  - An initally uniform flow hits a no-slip condition at the wall. To preserve mass, the interior region speeds up.
  - A boundary layer develops (B.L. separates region where viscous effects have been communicated, or felt).
  - Entry Length: $L_h/D = 0.05Re$ for laminar, and $L_h/D \approx 10$ for turbulent.
    - Must correct for laminar, but ignore for long pipes when turbulent.
  - Pressure drop greatest in the entry region.
- Velocity Profile
  - $u = u(r)$: 1-D problem.
  - Do a force balance on a cylindrical shell: Pressure and viscous forces.
  - Take limit as Δx and $\Delta r \to 0 \to$ a PDE.
    - $-\frac{\partial P}{\partial x} = \frac{1}{r}\frac{\partial r\tau}{\partial r}$.
  - Left side is f(x) and right side is f(r) so each side equals a constant.
    - That is, dP/dx is constant.
  - Solve for τ with B.C. $\tau = 0$ at $r = 0$.
    - $\tau = -\frac{dP}{dx}\frac{r}{2}$
  - Now use $\tau = -\mu\frac{du}{dr}$ (note the sign: τ is + to right, but du/dr is negative.
  - Insert and solve for u with B.C. $u = 0$ at $r = R$.
    - $u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$.
      - · Parabolic, $u(R) = 0$, $u(0) = u_{max}$.
      - · $u_{avg} = \frac{1}{A}\int_A u dA = -\frac{R^2}{8\mu}\frac{dP}{dx} = u_{max}/2$.
  - Note: u_{avg} is constant over length, so integrate over length:
  (**) $P_1 - P_2 = \Delta P = \frac{32\mu u_{avg}L}{D^2}$. (Note sign).
  - Recall $\tau = -\left(\frac{dP}{dx}\right)\frac{r}{2} \to \tau_w = -\left(\frac{dP}{dx}\right)\frac{R}{2} = -\left(\frac{dP}{dx}\right)\frac{D}{4}$.
    - Integrate over the length $\to 4\tau_w = \frac{\Delta P D}{L}$.
      - · TRADEOFF BETWEEN PRESSURE AND FRICTION.
  - Ratio of friction to inertia:
    - $\frac{4\tau_w}{\rho v^2/2} = \frac{8\tau_w}{\rho u_{avg}^2} = \frac{2\Delta P D}{L\rho u_{avg}^2} = f =$ the Darcy Friction factor.
      - · Works for laminar or turbulent.
  - For laminar, insert (**) for $\Delta PD/L \to f = 64/Re$.

Class 16 - Laminar Pipe Flow.

Chp 8 Takes us through next exam

- Laminar Flows (Today)
  - Re
  - Entrance Region
  - Equations
  - Pressure Drop / Friction Factor
- Turbulent Pipe Flow
- Minor losses - values, etc
- Pipe Networks
- Flow meters

- Internal Flows
- Pipe flows are of Central importance.

• S.S., Incompressible.

* Key Concept: Pressure Drop Balanced By friction

Reynolds

• Last class saw 2 Groups: $Re = \frac{\rho D V}{\mu}$, $\frac{\Delta P\, D}{L \rho V^2}$

• Flow Classified as Laminar or Turbulent.

- Laminar - smooth streamlines, ordered motion
- Turbulent - Chaotic, random, Disordered

• $\boxed{\text{Movie}}$ — Reynold Experiment - Dye in pipe, increase Re via Velocity

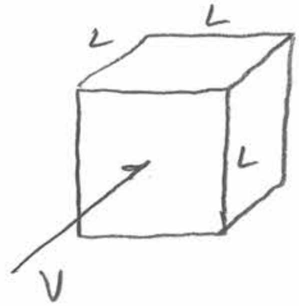
• Re Characterizes Flow: Lam ← 2300 - - - Trans - - - 4000 → turb

  • $Re = \frac{\rho D V}{\mu}$ or $\frac{DV}{\nu}$ or replace D with Characteristic Length Scale L.

  • Noncircular Ducts → $D_h = 4A_c / P_w$

What is the Re ?

- Ratio of inertial to viscous forces



$F = \text{mom}/\text{time}$

$F_{\text{Inertial}} = F_I = \frac{mV}{t} = \frac{\rho L^3 V}{L/V} = \rho L^2 V^2$

$F_{\text{visc}} = F_V = \tau \cdot A = 4L^2 \mu \frac{dV}{dx} \sim \frac{L^2 \mu V}{L} \rightarrow L\mu V$

$F_I / F_V = \rho L^2 V^2 / L\mu V = \rho L V/\mu = Re$

- Ratio of timescales : Diffusive / Convective
  - $\tau_D = L^2/\nu$ (see class 15)
  - $\tau_c = L/V$
  - $\tau_D/\tau_c = \frac{L^2}{\nu L/V} = \frac{LV}{\nu} = \frac{\rho L V}{\mu} = Re$
  - $\frac{1}{\tau}$ is a rate $\rightarrow$ Re is the rate of Convection / rate of Diffusion.
  - for $Re = 1000 \rightarrow \approx 1000$ times longer to Diffuse than to Convect, over L
- Can also write as Length Scale Ratio

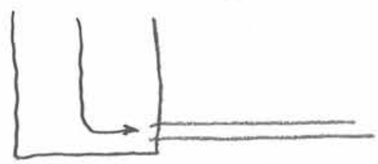
- High Re $\rightarrow$ Turbulent

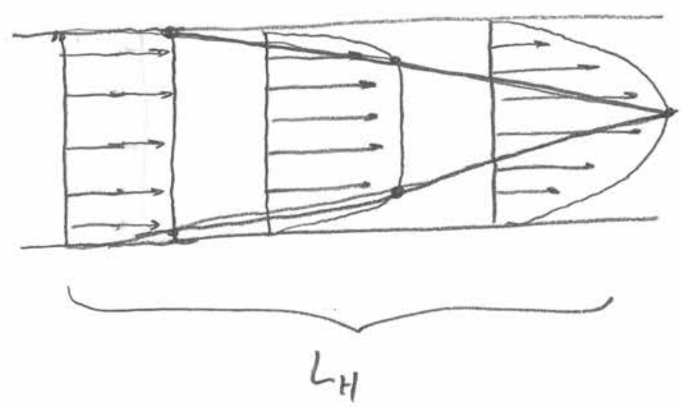
Q: Lot

		Re
Example :	Car @ 3MPh, 10 ft long $\rightarrow$	270,000
	Heater Vent @ 10mph, 6" $\rightarrow$	45,000
	faucet @ 1 m/s, 1/2" $\rightarrow$	13,000

Laminar Flow is The Exception, not The Rule

Entrance Region

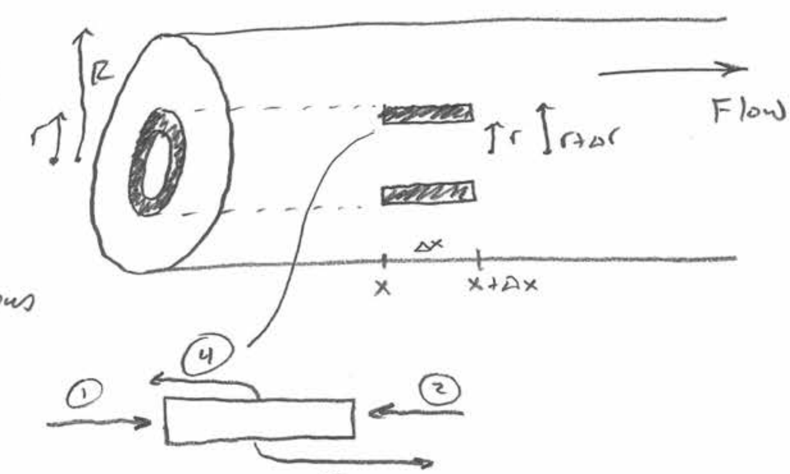




- Flow must Develop
- Requires time (space)
- Uniform Flow → no slip at walls
  - walls → 0
  - center increases
  - Boundary layers Develop until meet in center

- Laminar : $\frac{L_H}{D} = 0.05\,Re$ → $L_H = 10$ ft in 1 in Pipe for $Re = 2300$
- Turbulent : $\frac{L_H}{D} = \approx 10$ → Ignore for long Pipes

Velocity Profile. (Laminar)



1-D → $u = u(r)$

Force Balance – Pressure, Viscous

- S.S. → $\sum F = 0$

- $\underset{①}{(P_x} - \underset{②}{P_{x+\Delta x})}(2\pi r\Delta r) + \underset{③}{\tau_r(2\pi r\Delta x)} - \underset{④}{\tau_{r+\Delta r}(2\pi(r+\Delta r)\Delta x)} = 0$

- $\div\, 2\pi\Delta r\Delta x$, $\lim \Delta r \to 0,\ \Delta x \to 0$

$$\underbrace{-\frac{dP}{dx}}_{f(x)} = \underbrace{\frac{1}{r}\frac{d(r\tau)}{dr}}_{f(r)} \rightarrow\ = \text{Const} \rightarrow \frac{dP}{dx} \text{ is Const. (a "\#")}$$

$$\boxed{\frac{dP}{dx} = \text{Const}}$$

- Solve for τ with $\tau = 0$ at $r = 0$ as BC

$$\tau = -\frac{dP}{dx}\cdot\frac{r}{2} \longrightarrow \boxed{\tau \propto r}$$

$$\tau_w = -\frac{dP}{dx}\cdot\frac{R}{2} \longrightarrow \boxed{\tau_w = -\frac{\Delta P}{L}\cdot\frac{D}{4}} \longrightarrow \boxed{4\tau_w = \frac{-\Delta P D}{L}}$$

$$\left(\frac{dP}{dx} = c \rightarrow \frac{\Delta P}{L} = \frac{dP}{dx}\right)$$

- Insert $\tau = -\mu \frac{du}{dr}$, (note sign: τ is + to right, but du/dr is neg)

Solve for $u(r)$

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$$

- Parabolic
- $u(R) = 0$
- $u(0) = u_{max}$ $\longrightarrow$

$$\boxed{\begin{aligned} u_{max} &= -\frac{R^2}{4\mu}\frac{dP}{dx} \\ u_{max} &= -\frac{D^2}{16\mu}\frac{dP}{dx} = \frac{-D^2\Delta P}{16\mu L}\end{aligned}}$$

$$u_{avg} = \frac{1}{A}\int_A u(r)\,dA = \frac{-R^2}{8\mu}\frac{dP}{dx} = \boxed{\frac{u_{max}}{2} = u_{avg}}$$

$$\boxed{u_{avg} = \frac{-D^2\Delta P}{32\mu L}} \quad \boxed{\square}$$

- Recall: we had a Group $\left(\frac{\Delta P D}{L\rho V^2}\right) = \frac{-4\tau_w}{\rho V^2}$ using ◎ above

if we used $\frac{\Delta P D}{L\rho(V^2/2)}$ instead $\longrightarrow$ $\boxed{\frac{8\tau_w}{\rho V^2} = f_{Darcy}}$

(our 2 key Groups are Re, f) $\quad$ f is the friction factor

- For Laminar, insert ◎ into $\boxed{\square}$ for $-\Delta P\, D/L$

$$\longrightarrow \boxed{f = 64/Re}$$

- Pipe flow: $Re \longrightarrow f \longrightarrow \Delta P$; ΔP related to Flow!