ChE 374–Lecture 13–Mechanical Energy Analysis of Steady Flow

- Kinetic Energy Correction Factor
  - $\frac{v^2}{2}$ term appears in energy equations
  - Usually assume uniform v.
  - In general v is nonuniform (e.g., pipes have v=0 at the walls).
    - For mass flow, just use average velocity, then can treat as uniform.
    - For K.E. this does not work since $\bar{v}^2 \neq \overline{v^2}$
  - Correct with a fudge factor, α: $K.E. = \frac{1}{2}\alpha\bar{v}^2$.
  - $\alpha = 2$ for laminar flow and $\alpha = 1.04 - 1.11$ for turbulent flow.
  - Often ignored:
    - Most flows are turbulent
    - K.E. is small compared to ΔP, or Δz.
    - Small error compared to other assumptions or unknowns.
- Friction and Mechanical Energy Losses
  - SS Energy Equation:

$$\dot{Q} + \dot{W}_s = \dot{m}\Delta u + \underbrace{\dot{m}\left(\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2} + g\Delta z\right)}_{\Delta e_{mech}}.$$

  - We've been ignoring $\dot{Q}$, $\dot{W}_s$, Δu.
  - Now, account for $\dot{W}_s$ and "Losses".
  - Observations:
    1 Take $\dot{Q} = \dot{W}_s = \Delta u = 0 \to \Delta e_{mech} = 0 \to e_{mech}$ is conserved.
    2 Consider a heated pipe, no friction, then $\dot{m}\Delta u = \dot{Q}$.
      · So that heat $\to \Delta u$
    3 Friction converts mechanical energy to internal energy.
      · Δu not from heat transfer is from friction losses. Rearrange Energy Eqn:

$$\dot{m}\left(\frac{\Delta P}{\rho} + \frac{\Delta v^2}{2} + g\Delta z\right) = \dot{W}_s + \underbrace{(\dot{Q} - \dot{m}\Delta u)}_{-F}.$$

      · F is +.
      · F decreases e_{mech}.
      · Book calls this $\dot{E}_{mech,loss}$.
    4 Head Form:

$$\frac{\Delta P}{\rho g} + \frac{\Delta v^2}{2g} + \Delta z = h_w - h_L$$

      · with $h_L = F/\dot{m}g$, $h_w = \dot{W}_s/\dot{m}g$.
      · Normally F is friction in pipes, with losses in pumps accounted with efficiency.
- Examples:
  - Example 1: Raise a liquid: $h_w = \Delta z + h_L$.
  - Example 2: Pump a liquid: $h_w = \frac{\Delta P}{\rho g} + h_L$.
  - Example 3: Nozzle: $-\frac{\Delta P}{\rho g} = \frac{\Delta v^2}{2g} + h_L$.
  - In each case, h_L is wasted: Δz, ΔP, and Δv are less than without loss.
  - Text 5-94.
- Note: one equation, one unknown: ($\dot{m}$, ΔP, Δv^2, Δz, $\dot{W}_s$, η, F).
  - Set all but one and solve for that one. CONSIDER THIS IN BOOK EXAMPLES, HW.

Lecture 13 – Mechanical Energy Analysis of Steady Flow

Items: Class Project
K.E. correction factor α
Friction / Loss
Examples.

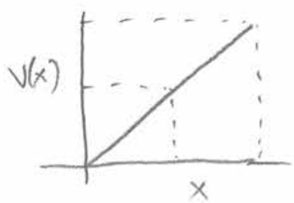
K.E. correction factor.

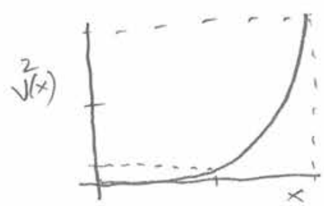
- $\frac{V^2}{2}$ Term appears
- Been assuming uniform V
- In general $V \neq$ uniform:

Pipes

- For mass flow, just use $\bar{V}$

- But for K.E. this does not work: $\bar{V}^2 \neq \overline{V^2}$





linear → $\bar{V} = V(\bar{x})$ nonlinear → $\overline{V^2} \neq \left(V(\bar{x})\right)^2$

- Correct w/ a fudge factor: $KE = \frac{1}{2}\alpha\bar{V}^2$

* $\alpha = 2$ for laminar pipe
$\alpha = 1.04 - 1.11$ for Turb. flow.

- often ignored.
  - most flows turb
  - KE small vs p or h
  - Small relative to other assumptions.

Friction / Losses

SS Energy Equation

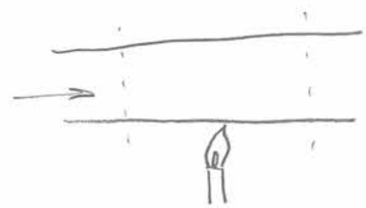
$$\dot{Q} + \dot{W}_s = \dot{m}\,\Delta u + \dot{m}\underbrace{\left(\frac{\Delta p}{\rho} + \frac{\Delta V^2}{2} + g\Delta z\right)}_{\Delta e_m}$$

- Been ignoring $\dot{Q}$, $\dot{W}_s$, Δu
- Now, add in $\dot{W}_s$, "Losses"

Observations

1. $\dot{Q} = \dot{W}_s = \Delta u = 0 \rightarrow \Delta e_m = 0 \rightarrow e_m$ conserved
   - B.E.

2. Heater, no friction,

   $\dot{m}\,\Delta u = \dot{Q}$

   Heat $\rightarrow \Delta u$

3. Friction converts mechanical energy to internal energy

   Δu not from heat transfer is friction Loss

$$\dot{m}\left(\frac{\Delta p}{\rho} + \frac{\Delta V^2}{2} + g\Delta z\right) = \dot{W}_s + \underbrace{\left(\dot{Q} - \dot{m}\,\Delta u\right)}_{-\tilde{\mathcal{F}}}$$

- $\tilde{\mathcal{F}}$ is +
- Decreases e_m
- Book calls this $\dot{E}_{mech,\,Loss}$

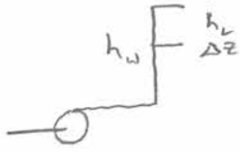
4. Head Form:

$$\frac{\Delta p}{\rho g} + \frac{\Delta V^2}{2g} + \Delta z = h_w - h_L \qquad ; \qquad h_L = \frac{\tilde{\mathcal{F}}}{\dot{m}\cdot g}$$

- normally this is Friction in pipes, with losses in pumps accounted with efficiency

Example - Raise a Liquid

$\boxed{\Delta z:}$



$$\cancel{\frac{\Delta P}{\rho g}} + \cancel{\frac{\Delta V^2}{2g}} + \Delta z = h_w - h_L$$

$$* \; h_w = \Delta z + h_L$$

Example - Pump a Liquid.

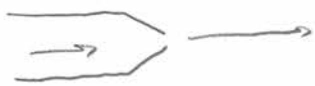
$\boxed{\Delta P:}$

$$\frac{\Delta P}{\rho g} + \cancel{\frac{\Delta V^2}{2g}} + \cancel{\Delta z} = h_w - h_L$$

$$* \; h_w = \frac{\Delta P}{\rho g} + h_L$$

Example - Nozzle

$\boxed{\Delta V}$



$$\frac{\Delta P}{\rho g} + \frac{\Delta V^2}{2g} + \cancel{\Delta z} = \cancel{h_w} - h_L$$

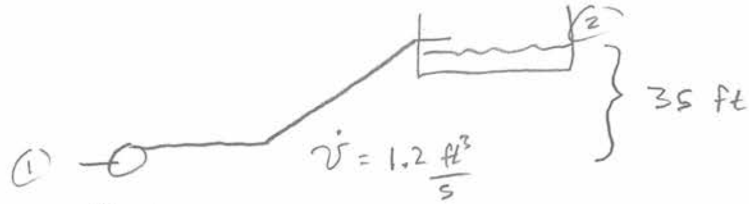
$$\frac{-\Delta P}{\rho g} = \frac{\Delta V^2}{2g} + h_L$$

- In each case h_L is wasted, Δz, ΔP, ΔV is less!

- $(W_s - F) = W_u = \dot{m}\Delta e_m$; W_u is the useful work!

Example

Text 5-94



$\eta = 0.73$

$\dot W = 12$ hp

- Find h_L,
- Find Power to overcome it.

$\boxed{\text{T.P.S.}}$ → • Q: where to start ? → E.E. → write it.

$$W_s - F = \dot m\left(\frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2\right) - \dot m\left(\frac{P_1}{\rho} + \frac{V_1^2}{2} + \cancel{gz_1}\right) \qquad z_1 = 0$$

• Pick 2 points : - with W_s, F The Locations matter
- Go w/ The Flow
- W_s raises e_m → 2 - 1 is Top - bot
- else get wrong signs.

• Simplify : $V_1 = V_2 = 0$ $\quad P_1 = P_2 = P_{atm}$

$z_2 = h$ $\quad \dot m = \dot m = C$

→ $\dot W_s - F = \dot m g h$

• Head Form $\div \dot m g$

→ $$\frac{\dot W_s}{\dot m g} - h_L = h$$

Terms: $\dot W_s = ? = \eta W = 8.76$ hp

$\dot m = \rho \dot V = (62.3\ lbm/ft^3)(1.2\ ft^3/s) = 74.76\ lbm/s$

$g = 32.2\ ft/s^2$

$h = 35\ ft$

→ $\boxed{h_L = 29.4\,ft}$

$\text{Power}_L = \dot m g h_L = \boxed{4.0\ \text{Hp}}$ ↗

$$\frac{\dot W_L}{\dot W_s} = \frac{h_L}{h + h_L}$$

→ $\underline{\dot W_L = 4.0\ hp}$

- Note: we could reverse all this too → Dam Power.
- Note 1 Eqn, 1 Unknown.

$$\begin{pmatrix} \dot{m} \\ \Delta P \\ \Delta V^2 \\ \Delta z \\ W_s \\ \eta \\ F \end{pmatrix}$$

- Set or assume all but one
- Solve.
- All prob are like This.
- Use when reading book examples.
- Note how to convert between head forms.