Chemical Engineering 374

Fluid Mechanics

Exam 1 Review

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Exam Review, By Content/Lectures

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- Classes 1-9 (plus review)
- Chapter 1 Introduction/Basics
- Chapter 2.1-2.6 Fluid Properties
- Chapter 3.1-3.6 Pressure/Fluid Statics
- Chapter 4.1, 4.6 RTT, Conservation Laws
- Chapter 5.1-5.5 M.B., E.B., Bernoulli
- Homeworks 1-9

Class 1--Introduction

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- Syllabus
- Schedule
- Policies
- Competencies/Goals
- Info Sheets
- Units, esp. lbm vs. lbf
- Sig. Figs.
- Problem Solving

Class 2 – Fluid Properties

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- Fluid definition in terms of stress
- Liquids vs gases
- Continuum assumption
- Density – gases, liquids (T, P variations)
  - Coefficient of compressibility (P, V)
  - Coefficient of volume expansion (T, V)
- Viscosity– in terms of stress ($\tau=\mu$ du/dy)
  - Force per area, also a momentum flux.
- Non-Newtonian Fluids
  - Newtonian
  - Bingham plastic
  - Pseudoplastic
  - Dilitant

Class 3 – Pressure/Fluid Statics

- Pressure (F/A), a normal stress, isotropic/scalar.
  - Units
  - Absolute vs gage (which/when/specify)
- Barometric equation $\frac{dP}{dz} = -\rho g$
  - Force balance on a body, divergence theorem

    $\sum \vec{F} = 0 = \vec{F}_p + \vec{F}_b \quad \vec{F}_p + \vec{F}_b = -\int_{SA} P\vec{n}dA + \int_V \rho\vec{a}dV \quad \vec{F}_p + \vec{F}_b = -\int_V \nabla P dV + \int_V \rho\vec{a}dV$
  - $\Delta P = \rho g h$ (for constant density).
  - Know what to do for variable ρ
    - Ideal Gas: Isothermal → ideal gas law
    - Ideal Gas: Adiabatic → ideal gas law with T, P relation, where $\gamma = c_p/c_v \sim$ const

$$T = \frac{T_1}{P_1}^{\alpha} P^{\alpha} \qquad \alpha = \frac{\gamma - 1}{\gamma}$$

  - Liquid P variation with depth, gas variation with depth.

Class 4—Pressure Measurement/Surface Forces

- Barometer (P_{atm})
- Bourdon Tube (gage pressure)
- Manometer (pressure differences)
  - Pressure is the same at the same height in a continuous fluid.
  - Travel from one end to the other. "down" increases pressure, "up" decreases pressure.
    - Gas/Liquid calculation: delta P = = ρ * g * h
    - Liquid/Liquid calculation: delta P = $\Delta\rho$ * g * h

Class 4—Pressure Measurement/Surface Forces

- Forces on surfaces
  - F = P*A → dF = P*dA → (often) dF = P*W*dh → Integrate to get net force
  - Net force is Area * P at the centroid (area-weighted average depth).
  - Point of exertion of that force is through the "center of pressure" (force-weighted average depth, or the centroid of the "pressure prism").
  - For a flat plate, centroid is the middle of the plate, and the pressure center is 2/3 down the plate.
  - Atmospheric pressure cancels when acting on both sides
  - Inclined surfaces are just like vertical, but have an angle between dA and dh
- Bouyancy
  - Net pressure force on a submerged body.
  - Force is upward equal to the weight of displaced fluid.
  - Can neglect bouyancy if account for forces (body/pressure) as before (or rather, we are accounting for it directly).

Class 5—Math/RTT

- Scalar, vector, tensor (basic idea only).
- Dot product
- Mass flux = $\rho A\mathbf{v}\ .\ \mathbf{n}$ = $\rho Av^*\cos\theta$
- Lagrangian system: follow some fixed mass: moves/deforms
  - No mass crosses boundary.
  - Conservation laws defined for Lagrangian systems
- Eulerian control volume: consider some (fixed) space
  - Convenient for engineering analysis
- Material derivative/substantial derivative
  - (Rate of change following the flow) = (rate of change at a fixed point) + (rate of change due to change with position)

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}$$

Class 5—Math/RTT

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- Reynolds Transport Theorem
  - Relates a Lagrangian system for which a conservation law applies, to an Eulerian Control Volume, which we want a conservation law for.

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} \rho b dV + \int_A \rho b \vec{v}\cdot\vec{n} dA$$

  - B is mass, b = 1 for mass conservation.
  - B is total energy E, b = total energy per mass for energy conservation.
  - When in doubt (especially when things are not simple/uniform), start with this equation and simplify.
  - Usually apply for fixed control volumes, but works for variable control volumes too.
  - Related to the substantial derivative, but the S.D. is not finite like the integral form, so it has no deformation component.

Class 6—Mass Balance

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- RTT → B = M, b = 1.
- Conservation of mass → dB/dt = 0.
- RTT → $0 = \frac{d}{dt}\int_{CV} \rho dV + \int_A \rho \vec{v}\cdot\vec{n} dA$
- Know how to use this equation in terms of various assumptions.
  - Steady, constant volume, constant density, single streams.
- Steady state → drop the first term
- Uniform within the control volume: first term → dM/dt
- Single inlet and outlet, uniform ρ, average v across outlet: second term → $\dot{m}_{out} - \dot{m}_{in}$
- Density for ideal gases: ρ=MP/RT
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Class 7—Integral Energy Balance

- RTT → B=E, b=e, e = u+v^2/2+gz
- Conservation law: dE/dt = dQ/dt + dW/dt
  - 1^{st} law of thermodynamics

$$\frac{dQ}{dt} + \frac{dW}{dt} = \frac{d}{dt}\int_{CV} \rho(u + \frac{1}{2}v^2 + gz)dV + \int_{CS} \rho(u + \frac{1}{2}v^2 + gz)\vec{v} \cdot \vec{n} dA$$

  - Work is shaft and flow work. Split these up, and also assume uniform properties and single streams. u+P/ρ = h

$$\frac{dQ}{dt} + \frac{dW_s}{dt} = \frac{d}{dt}\left[\rho(u + \frac{1}{2}v^2 + gz)V\right] + \left[\rho v A(u + \frac{P}{\rho} + \frac{1}{2}v^2 + gz)\right]_{out} - []_{in}$$

  - Know how to simplify the problem for various assumptions
    - Steady state, constant density, shaft work, etc.
- Mechanical energy
- Efficiency

Class 8—Bernoulli Equation

$$\Delta\left(\frac{P}{\rho} + \frac{1}{2}v^2 + gz\right) = 0$$

- Mechanical energy is conserved
- Recall assumptions
  - SS, frictionless, const ρ, no W_s, no Q, streamline
- Energy form, pressure form, head form
- Pitot tubes → velocity measurement
- Grade lines (head form)
- Version across streamlines, compressible

Class 9—Bernoulli Applications

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- Problem solving approach
- Tank problem
- Water faucet problem
- Convert between P, v, z
- Often mix mass and energy balances to find desired quantities.
- How to pick the control volume.

How to Study and Solve Problems

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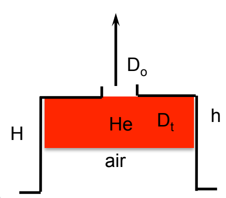
- Start Early
- Understand concepts, including key assumptions
- Know how to apply these concepts.
- Practice: book examples, homework
- Most problems involve one key concept, with auxiliary concepts
  - Bernoulli equation,
    - But need to know key mass balance relations
      - mdot=ρ*A*v, or Vdot = v*A or $v_1A_1=v_2A_2$
    - And unit conversions
    - And diameter to area: $A = \pi D^2/4$
- Be really organized
  - Put in numbers last.
  - Keep everything in a column instead of everywhere.
  - Use scratch paper for your scratching.

Sample Exam Problem

Helium (He) flows out of a round tank through a round hole in the top.
The tank is open to the atmosphere at the outlet hole and at the bottom of the tank.
As He flows out the top, air fills in from the bottom (but doesn't mix with the He).
The hole diameter, tank diameter, and tank height are $\mathbf{D_o = 2\ cm}$, $\mathbf{D_t = 20\ cm}$, $\mathbf{H = 1\ m}$.
Initially, the tank is full of He, so h=H (h is the height of the helium column).
Take $\mathbf{MW_{He} = 4}$, $\mathbf{MW_{air} = 29\ kg/kmol}$.
The tank is considered so much bigger than the hole, that at any instant the system is At steady state.

(a) Find the initial velocity of He through the hole.
(b) Find the time for all the He to leave the tank.



- Read carefully.
- Relate words to system and assumptions
- Break it up.
- What are the key principles involved?
- How do they relate to my unknowns?
- Do numbers last.

All the Textbook You'll Ever Need (Exam 1)

Properties

$\rho_{air} = 1.204\ kg/m^3$ @ 20°C, 1atm
$\mu_{air} = 1.825 \times 10^{-5}\ kg/m \cdot s$

$\rho_w = 998\ kg/m^3$ 20°, 1 atm
$\mu_w = 1.002 \times 10^{-3}\ kg/m \cdot s$
$\rho_w = 62.3\ lbm/ft^3$

$g = 9.81\ m/s^2 = 32.2\ ft/s^2$
$1\ atm = 14.7\ psi = 101325\ Pa$

Units

Force (=) N (=) $kg\,m/s^2$
Energy (=) J (=) $kg\,m^2/s^2$
Power (=) W (=) $kg\,m^2/s^3$
Pressure (=) Pa (=) $kg/m s^2$

μ (=) $\frac{kg}{m \cdot s}$
ν (=) $\frac{m^2}{s}$

* $F = m \cdot a \rightarrow (1\ lbm) \times 32.2\ ft/s^2 = 32.2\ lbm\ ft/s^2 \cdot \frac{lbf}{32.2\ lbm\ ft/s^2} = (1\ lbf)$

Defs

$SG = \frac{\rho}{\rho_{H_2O}}$ S.W. $\gamma = \rho g$ $\rho = \frac{MP}{RT}$; $M_{air} = 29$; $M_{H_2O} = 18$

$A = \frac{\pi}{4} D^2$; $V = \frac{\pi}{6} D^3$

$\tau = F/A$, $P = F/A$; $\tau = \mu \frac{du}{dy}$

$P_g = P_{abs} - P_{atm}$

$\Delta P = \rho g h$; $\frac{dP}{dz} = -\rho g$; $\nabla P = \rho \vec{a}$

$F = PA \rightarrow dF = P dA \rightarrow F = \int P dA = \int \rho g h\, dA$

- Centroid: $Y_c = \frac{1}{A} \int y\, dA$. pressure center
- Buoyancy: F_b = Displaced Fluid weight : F_b = weight object, F_b from $P \rightarrow$ ...

$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{V} \cdot \nabla$ $\frac{dB_{sys}}{dt} = \frac{d}{dt} \int_{C.V.} \rho b\, dV + \int_{C.S.} \rho b \vec{V} \cdot \vec{n}\, dA$ S.S. Const ρ Unif Fixed C.V.

M.B. $\frac{d}{dt} \int_{C.V.} \rho\, dV + \int_{C.S.} \rho \vec{V} \cdot \vec{n}\, dA = 0 \rightarrow \frac{dm_{C.V.}}{dt} = \dot{m}_{in} - \dot{m}_{out}$

E.B. $\frac{dQ}{dt} + \frac{dW_s}{dt} = \frac{d}{dt} \int_{C.V.} \rho \left(u + \frac{V^2}{2} + gz\right) dV + \int_{C.S.} \rho \left(u + \frac{P}{\rho} + \frac{V^2}{2} + gz\right) \vec{V} \cdot \vec{n}\, dA$

B.E. $\frac{\Delta P}{\rho} + \frac{\Delta V^2}{2} + g \Delta z = 0$

– Energy, pressure, head forms.