Chemical Engineering 374
Non-Newtonian Flow Homework Assignment

1. Derive the following equation describing the velocity profile of a Power-Law Fluid in laminar flow in a horizontal cylindrical pipe

$$v = \left(-\frac{1}{2K}\frac{dP}{dx}\right)^{1/n}\left(\frac{n}{n+1}\right)\left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right)$$

Begin with τ =-(r/2)*dP/dx and substitute in

$$\tau = K\left(-\frac{dv}{dr}\right)^{n}$$

for the shear stress. Then integrate to obtain velocity and apply the no-slip boundary condition to evaluate the integration constant.

2. Derive the following equation describing the volumetric flow rate of a Power-Law Fluid in laminar flow in a horizontal cylindrical pipe.

$$Q = \frac{\pi n D^3}{8(3n+1)}\left(-\frac{D}{4K}\frac{dP}{dx}\right)^{1/n}$$

3. A polymer melt is to be pumped at a flow rate of 8.3x10^{-4} m^3/s through 2100 *m* of 3.5-*cm*-ID pipe from the bottom of one open tank to the bottom of another open tank 14.7 *m* higher in elevation. The polymer melt has a density of 814 kg/m^3, and it's rheology seems to be well described by the relation:

$$\tau = .045\frac{kg}{m\ s^{1.22}}\left(\frac{dv}{dz}\right)^{0.78}$$

Assuming laminar flow, how much power must the pump deliver?

4. You suspect that the fluid for which you must design a pumping system is non-Newtonian and can be described by a Power-Law model. Acting on that suspicion, you assigned a junior engineer to take some data for that fluid in a couple of horizontal cylindrical pipes* already set up in your facility. The engineer brings you the following data:

Pipe Diameter** (*inches*)	Flowrate (ft^3/min)	Pressure Loss* (*psi*)
1.0	0.075	4.9
1.0	0.20	10.0
1.0	0.41	17.0
1.0	0.68	24.6
1.0	1.00	32.5
1.0	1.36	40.9
1.5	5.75	32.2
1.5	6.79	36.8
1.5	7.36	41.3
1.5	7.71	46.5
1.5	8.11	52.7
1.5	8.33	59.0

*The length of both pipes is 280 *feet*.
**These are the actual diameters, since the pipes in the facility are copper pipes, not industrial steel pipes.

a) Does the fluid appear to be a non-Newtonian Power-Law fluid? If so, which does it appear to be: pseudoplastic or dilatant?

b) Which data points represent laminar flow?

c) What value of "*n*" would be most appropriate for the Power-Law model?

d) What value and units of "*K*" would be most appropriate for the Power-Law model? (Note: the units of *K* depend on the value of *n*)

e) If the fluid has a density of 60 lb_m/ft^3 and is not a turbulence-reducing fluid, what friction factor would be appropriate for the highest flowrate given above?

f) How much power would be needed to pump this polymer from one tank to another (both open to the atmosphere at the same height) through 500 feet of copper pipe (1.5-inch diameter) at 8.33 ft^3/min? Assume the conditions given in part e).

Hint: You can use whatever tools you like, but I strongly recommend a spreadsheet.