## Exercise

- Make a list of all python functions/variables/code we have seen.
- Discuss the items in the list:
- What is the basic idea?
- What further details?
- What are some "gotchas" or caveats?


## Excercise

- Write down the main topic/title of the last five classes on Python
- For each class, what were the main ideas
- Outline each class.
- Put in details
- What examples were done?
- Do this alone, then with your neighbor.
- Create your own review notes (like these slides).
- First "recall" then look up to fill in.


## Class 20: Examples, nonlinear solvers

- Discussed Newton Fractal
- Discussed turbulent problem: coupling nonlinear solve
- Discussed homework
- Adiabatic flame temperature solution
- Approaches for solving systems of equations


## Solving nonlinear equations

Args/tuple expansion, nonlinear HW, examples

## Interpolation

Curve fitting

- Use intermediate variables
- $R e=r D v / m$ normally doesn't need to be a separate equation/variable
- Often, the solution in Python can be written as it appears in text
- Carefully consider how many equations/variables you really need.
- Trivial equations can be used directly: like $\mathrm{x}_{1}+\mathrm{x}_{2}=1 \rightarrow \mathrm{x}_{2}=\left(1-\mathrm{x}_{1}\right)$
- In such cases, final values can be recovered after solution.
- Functions can be defined inside functions if convenient
- Can mix solution approaches: quad, interp1d, etc.


## Class 21: Interpolation

- Given a list of $x g$, $y g$ points.
- Find values of $y$ at intermediate $x$
- Linear interpolation exercise
- Pass in xg, yg, xw $\rightarrow$ find yw
- Find location (two bounding points).
- Form and evaluate equation for line between two points.
- Python interpolator

```
from scipy.interpolate import interp1d
xg = np.array([0,1,2,3,4,5,6,7,8,9,10])
yg = np.cos(xg**2.0/8.0)+1
f_interp = interp1d(xg, yg)
xw = 2.5
yw = f_interp(xw)
```


## Class 22: Curve Fitting

- Given a list of xg , yg points.
- Given a model function with unknown parameters
- Find the parameters
- polyfit, polyval functions

```
import numpy as np
xg = np.array([0., 1., 2., 3., 4., 5.])
yg = np.array([0, 0.8, 0.9, 0.1, -0.8, -1.0])
p3 = np.polyfit(xg, yg, 3)
xw = np.linspace(0,5,1000)
yw = np.polyval(p3, xw)
```


## Class 22: Curve Fitting

- Given a list of $x g$, yg points.
- Given a model function with unknown parameters
- Find the parameters
- curve_fit function

```
import numpy as np
from scipy.optimize import curve_fit
#-------- given data
xg = np.linspace(0,4,50)
yg = 2.5*np.exp(-1.3*xg)+0.5 + 0.2*np.random.normal(size=len(xg))
#-------- Define the function with parameters: x comes first
def f(x, a, b, c) :
    return a*np.exp(-b*x) + c
#-------- Do the curve fit
abc, extras = curve_fit(f, xg, yg)
a = abc[0]
b = abc[1]
c = abc[2]
xx = xg
yy = f(xx,a,b,c)
```


## Class 23: Rate Equations

- from scipy.integrate import odeint

$$
\frac{d y}{d t}=f(y, t) \quad y(0)=y_{0}
$$

- $f(y, t)$ is the "right hand side function" or the rate function.


## Rate equations

- $f(y, t)$ depends on $y$ and $t$ in general, but the actual expression often doesn't include $t$.
Symbolic math


## Widgets

## Python-Excel interface

- For multiple equations $y$ is a vector of "variables", $f$ is a vector of functions
- Solve for $\mathrm{y}(\mathrm{t})$. Solution will be an array of t and an array of y

$$
\begin{aligned}
& \frac{d y}{d t}=-2 y+3 \\
& y(0)=1 \\
& \text { def } \begin{aligned}
& f(\mathrm{y}, \mathrm{t}): \\
& \text { return }-2 * \mathrm{y}+3 \\
\mathrm{y} 0 & =1 \\
\mathrm{t} & =\text { np. linspace }(0,5,100) \\
\mathrm{y} & =\text { odeint }(\mathrm{f}, \mathrm{y} 0, \mathrm{t})
\end{aligned}
\end{aligned}
$$

$\frac{d v}{d t}=g$
$\frac{d x}{d t}=v$
$y(0)=x(0)=0$
def $f(v x, t)$ :
$v=v x[0]$
$\mathrm{x}=\mathrm{vx}$ [1]
$d v d t=9.81$
$d x d t=v$
return np.array([dvdt, dxdt])
xy0 = np.array ([0, 0])
$\mathrm{t}=\mathrm{np}$. linspace $(0,5,100)$
$y=\operatorname{odeint}(f, x y 0, t)$

